Surendranath Evening College
Department of Physics

OPTICS

PHS-A-CC-2-4-TH

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UNIT 4: INTERFERENCE OF LIGHT WAVES

Structure

4.1 INTRODUCTION

In 1680 Huygens proposed the wave theory of light. But at that time, it was not clear about the nature of light wave, its speed and way of propagation. In 1801 Thomas Young performed an experiment called Young's double slit experiment and noticed that bright and dork fringes are formed which is called inference pattern. At that time it was a surprising phenomenon and is to be explained.

After the Maxwell's electromagnetic theory it was cleared that light is an electromagnetic wave. In physics, interference is a phenomenon in which two waves superimpose on each other to form a resultant wave of greater or lower or of equal amplitude. When such two waves travel in space under certain conditions the intensity or energy of waves are redistributed at certain points which is called interference of light and we observe bright and dark fringes.

4.2 OBJECTIVES

After reading this unit you will able to understand

- The wave nature of light
- Phase and phase changes in light wave
- Coherence and coherent source of light
- Principle of superposition
- Young's double slit experiment and explanation
- Interference
- Interference phenomena in biprism and thin sheets

4.3 WAVE NATURE OF LIGHT

Light wave is basically an electromagnetic wave. Electromagnetic wave consists of electric and magnetic field vectors. The directions of electric and magnetic vectors are perpendicular to direction of propagation as shown in the figure 4.1. The electric and magnetic vectors are denoted by E and H and vary with time.

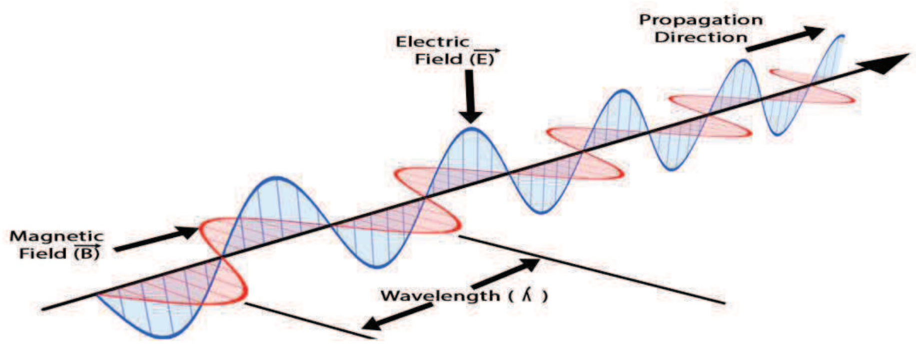


Figure 4.1

In light, electric vectors (or magnetic vectors) vary in sinusoidal manner as shown in figure 4.1. Therefore the electric vectors can be given as

$$E= E_0 \sin (kz - \omega t)$$

Where E = Electric field vector, E_0 = maximum amplitude of field vector, k = wave number (= $2\pi/\lambda$), z = displacement along the direction of propagation (say z axis), ω = angular velocity and t = time.

Before understanding the interference we should understand some terms and properties of light which are related to interference.

4.3.1 Monochromatic Light

The visible light is a continuous spectrum which consist a large number of wavelengths (approximately 3500Å to 7800Å). Every single wavelength (or frequency) of this continuous spectrum is called monochromatic light. However, the individual wavelengths are sufficiently close and indistinguishable. Some time we consider very narrow band of wave lengths as monochromatic light.

Ordinary light or white light, coming from sun, electric bulb, CFL, LED etc. consists a large number of wave lengths and hence non-monochromatic. But some specific sources like sodium lamp and helium neon laser emit monochromatic lights with wave lengths 589.3 nm and 632.8 nm respectively. It should be noted that sodium lamp, actually emits two spectral lines of wavelengths 589.0 nm and 589.6 nm which are very close together, and source is to be consider monochromatic.

4.3.2 Plane Wave

A plane wave is a wave whose wave front remains in a plane during the propagation of wave. In light wave, the maximum amplitude of electric vector E_0 remains constant and confined in a plane perpendicular to direction of propagation. Such type of wave called plane wave.

4.3.3 Polarized and Unpolarized Light

Light coming from many sources like sun, flame, incandescent lamp produce unpolarized light in which electric vector are oriented in all possible directions perpendicular to direction of propagation. But in polarized light electric vector are confined to only a single direction. The detail about polarized light will be discussed in the next block.

4.3.4 Phase Difference and Coherence

Wave is basically transportation of energy by mean of propagation of disturbance or vibrations. In wave motion through a medium, the particles of medium vibrate but in case of electromagnetic wave the electric or magnetic vectors vibrate form its equilibrium position.

The term phase describes the position and motion of vibration at any time. For example if y= a sin (ωt +θ) represents a wave, then the term (ωt +θ) represents the phase of wave. The unit of phase is degree or radium. After completion of 360^0 or 2π, the cycle of wave or phase repeats.

Phase difference

If there are two waves have some frequency then the phase difference is the angle (or time) after which the one wave achieves the same position and phase as of first wave. In the figure 4.2, two waves with phase different θ are shown.

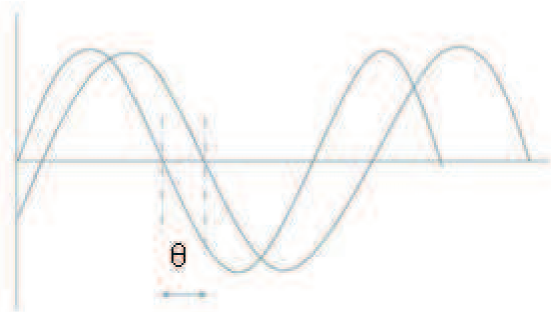


Figure 4.2

Coherence

If two or more waves of same frequencies are in same phase or have constant phase difference, those waves are called coherent wave. Figure 4.3 shows coherent wave with same phase (zero phase difference) and with constant phase difference.

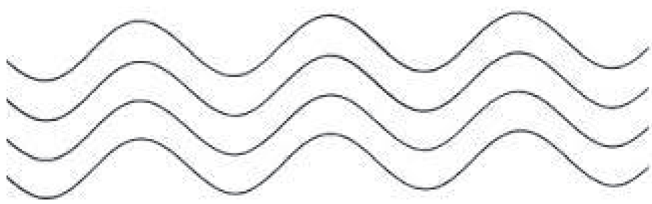


Figure 4.3

4.3.5 Optical path and Geometric Path

Optical path length (OPL) denoted by Δ is the equivalents path length in the vacuum corresponding to a path length in a medium. Path length in a medium can be considered as geometric path length (L). Suppose a light wave travels a path length L in a medium of refractive index μ and velocity of light is v in this medium, then for a time period t the geometric path length L is given by

$$L = vt$$

In the same time interval t, the light wave travel a distance Δ in vacuum which is optical path length corresponding to length L. Then

$$\Delta = ct = c\frac{L}{v}$$

Where, c is the velocity of light in vacuum.

or $$\Delta = \mu L$$

or The Optical path length = μ × (Geometrical path length in a medium).

In case of interference we always calculate optical path for simplification of understanding and mathematical calculations.

4.4 PRINCIPLE OF SUPERPOSITION

According to Young's principle of superposition, if two or more waves are travelling and overlap on each other at any point then the resultant displacement of wave is the sum of the displacement of individual waves (figure 4.4). If two waves are represented by $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t+\delta)$. Then according to principle of superposition, the resultant wave is represented by $y = y1 + y2$

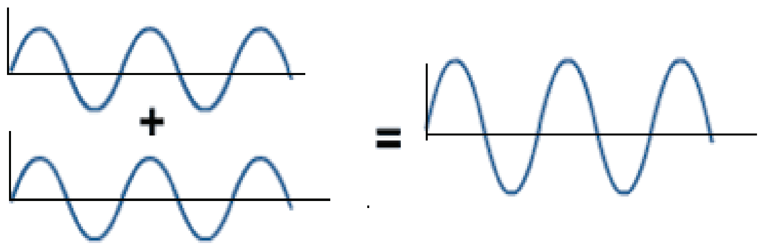


Figure 4.4

4.5 INTERFERENCE

When two light waves of some frequency, nearly same amplitude and having constant phase difference travel and overlap on each other, there is a modification in the intensity of light in the region of overlapping. This phenomenon is called interference.

The resultant wave depends on the phases or phase difference of waves. The modification in intensity or change in amplitude occurs due to principle of superposition. In certain points the two waves may be in same phase and at such point the amplitude of resultant wave will be sum of amplitude of individual waves. Thus, if the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 + a_2$. In this case, the intensity of resultant wave increases ($I \propto a^2$) and this phenomena is called constructive interference. Corresponding to constructive interference we observe bright fringes.

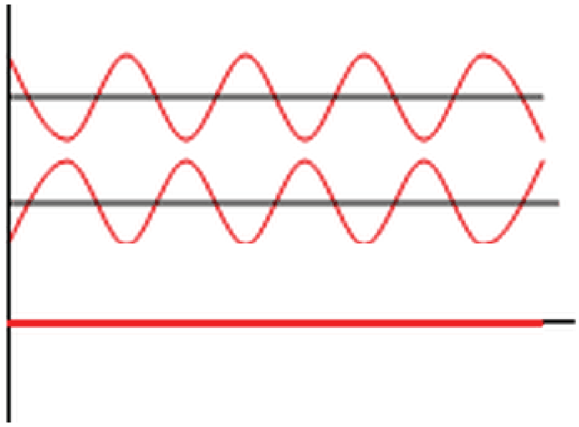


Figure 4.5

On the other hand, at certain points the two waves may be in opposite phase as shown in figure 4.4. In these points the resultant amplitude of waves will be sum of amplitude of individual waves with opposite directions. If the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 - a_2$ and the intensity of resultant wave will be minimum. This case is called destructive interference. Corresponding to such points we observe dark fringes. Figure 4.5 depicts two waves of opposite phase and their resultant.

4.5.1 Theory of Superposition

Let us consider two waves represented by $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t + \delta)$. According to Young's principle of superposition the resultant wave can be represented by

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta)$$

$$= (a_1 + a_2 \cos \delta) \sin \omega t + (a_2 \sin \delta) \cos \omega t \quad \text{....... (4.1)}$$

Let $$a_1 + a_2 \cos \delta = A \cos \emptyset \quad \text{....... (4.2)}$$

and $$a_2 \sin \delta = A \sin \emptyset \quad \text{....... (4.3)}$$

Where A and Ø are new constants, then above equation becomes

$$y = A \cos \emptyset \sin \omega t + A \sin \emptyset \cos \omega t$$

or $$y = A \sin (\omega t + \emptyset) \quad \text{....... (4.4)}$$

This is the equation of the resultant wave. In this equation y represents displacement, A represents resultant amplitude, $\emptyset$ is the phase difference.

From equation (4.2) and (4.3) we can determine the constant A and Ø. Squaring and adding the two equations, we get,

$$A^2 = a_1^2 + a_2^2 \cos^2 \delta + 2 a_1 a_2 \cos \delta + a_2^2 \sin^2 \delta$$

or $$A^2 = a_1^2 + a_2^2 + 2\, a_1 a_2 \cos\delta \qquad \text{....... (4.5)}$$

On dividing equation (4.3) by eq (4.2), we obtain,

$$\frac{\sin\emptyset}{\cos\emptyset} = \tan\emptyset = \frac{a_2 \sin\delta}{a_1 + a_2 \cos\delta} \qquad \text{....... (4.6)}$$

4.5.2 Condition for Maxima or Bright Fringes

If $\cos\delta = +1$ then $\delta = 2n\pi$ where $n = 0, 1, 2, 3……$(positive integer numbers).

Then, $$A^2 = a_1^2 + a_2^2 + 2a_1a_2 = (a_1+a_2)^2$$

Intensity, $$I = A^2 = (a_1+a_2)^2 \qquad \text{....... (4.7)}$$

Therefore, for $\delta = 2n\pi = 0, 2\pi, 4\pi……$, we observe bright fringes.

In term of path difference Δ

$$\Delta = \frac{\lambda}{2\pi л} \times \text{phase difference} = \frac{\lambda}{2\pi л}\, 2n\pi$$

or $$\Delta = n\lambda = \lambda, 2\lambda, 3\lambda \ldots \text{ etc.} \qquad \text{....... (4.8)}$$

4.5.3 Condition for Minima or Dark Fringes

If $\cos\delta = -1$ or $\delta = (2n-1)\pi = \pi, 3\pi, 5\pi \ldots\ldots$

Then $$A^2 = a_1^2 + a_2^2 - 2\, a_1a_2 = (a_1-a_2)^2$$

Intensity, $$I = A^2 = (a_1 - a_2)^2 \qquad \text{....... (4.9)}$$

Therefore if phase difference between two waves is $\delta = (2n-1)\pi = 0, 3\,\pi, 5\,\pi\ldots$ etc. is the condition of minima or dark fringes.

Now path difference, $\Delta = \frac{\lambda}{2\pi} \times Phase\ difference$

or $$\Delta = \frac{\lambda}{2\pi} \times (2n-1)\pi = \frac{(2n-1)}{2}\lambda = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots \qquad \text{....... (4.10)}$$

Example 4.1. Two coherent resources whose intensity ratio is 81:1 produce interference fringes. Calculate the ratio of maximum intensity and minimum intensity.

Solution: If I_1 and I_2 are intensities and a_1 and a_2 are the amplitudes of two waves then

$$\frac{I_1}{I_2} = \frac{81}{1} \ or\ \frac{a_1^2}{a_2^2} = \frac{81}{1} \ or\ \frac{a_1}{a_2} = \frac{9}{1}$$

Maximum intensity = $a_1+a_2 = 9\,a_2 + a_2 = 10\,a_2$

Minimum intensity = $a_1 - a_2 = 9\,a_2 - a_2 = 8\,a_2$

The ratio of maximum intensity to minimum intensity

$I_{max}/ I_{min}= (a_1+a_2)^2 / (a_1-a_2)^2 = 10^2/ 8^2 = 100/64=25/16$

4.5.4 Intensity Distribution

The intensity (I) of a wave can be given as $I = (½) \in_o a^2$ where a is the amplitude of wave, and $\in_0$ is the permittivity of free space. If we consider two waves of amplitudes a_1 and a_2 then at the point of maxima

$$I_{max} = (a_1+a_2)^2 = a_1^2+a_2^2+2a_1a_2$$

If $a_1 = a_2 = a$ then $I = 4a^2$. Therefore, at maxima points the resultant intensity is more than the sum of intensities of individual waves.

Similarly the intensity at points of minima

$$I_{min}= a_1^2+a_2^2 - 2a_1a_2 = (a_1\sim a_2)^2$$

If $a_1= a_2=a$ then $I_{min}=0$. Thus the intensity at minima points is less than the intensity of any wave.

The average intensity I_{av} is given as

$$I_{av}=\frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta} = \frac{\int_0^{2\pi} (a_1^2+a_2^2+2a_1\ a_2\ Cos\ \delta)d\delta}{\int_0^{2\pi} d\delta} = \frac{(a_1^2+a_2^2)2\pi л}{2\pi л} = a_1^2 + a_2^2$$

If $a_1 = a_2 = a$ then $I_{av} = 2a^2 = 2I$

Therefore, in interference pattern energy (intensity) $2a_1a_2$ is simply transferred from minima to maxima points. The net intensity (or average intensity) remains constant or conserved.

4.6 CLASSIFICATION OF INTERFERENCE

The interference can be divided into two categories.

4.6.1 Division of Wavefront

In this class of interference, the wave front originating from a common source is divided into two parts by employing mirror, prisms or lenses on the path. The two wave front thus separated traverse unequal paths and are finally brought together to produce interference pattern. Examples are biprism, Lloyd's mirror, Laser etc.

4.6.2 Division of Amplitude

In this class of interference the amplitude or intensity of incoming beam divided into two or more parts by partial reflection and refraction. Examples are thin films, Newton's rings, Michelson interferometer etc.

4.7 YOUNG'S DOUBLE SLIT EXPERIMENT

In 1801, Thomas Young performed double slit experiment in which a light first entered through a pin holes, then again divided into two pinholes and finally brought to superimpose on each other and obtained interferences. Young's performed experiment with sum light. Now the experiments are modified with monochromatic light and efficient slits.

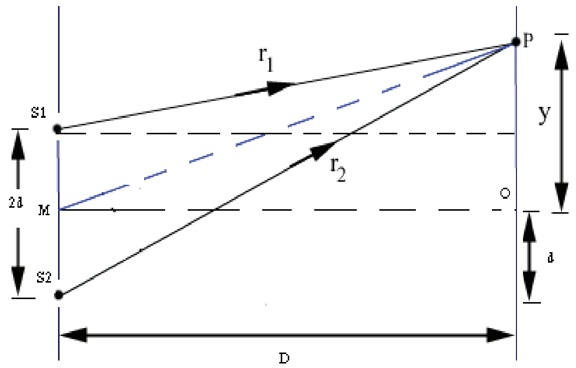


Fig. 4.6

Figure 4.6 shows the experimental setup of double slit experiment. S_1 and S_2 are two narrow slits illuminated by a monochromatic light source. The distance between two slits S_1 and S_2 is 2d. The two waves superimposed on each other and fringes are formed on the screen placed at a distance D from the centre of slits M. Let us consider a point P on the screen which is y distant from O. The two rays S_1P and S_2P meet at point P and produce interference pattern on screen.

Mathematically, path difference between rays S_1 P and S_2P is given as

$$\Delta = S_2P - S_1P \quad \text{....... (4.11)}$$

$$S_2P^2 = D^2 + (y+d)^2 = D^2[1 + (y+d)^2 / D^2]$$

$$S_2P = D[1 + (y+d)^2 / D^2]^{1/2}$$

$$= D[1 + \tfrac{1}{2}(y+d)^2 / D^2] \quad [\because (1+x)^n = 1 + nx + \cdots ...]$$

or

$$S_2P = D + (y+d)^2 / 2D \quad \text{....... (4.12)}$$

Similarly

$$S_1P^2 = D^2 + (y-d)^2$$

$$S_1P = D\,[1+ (y-d)^2 / D^2]^{1/2}$$

$$= D\,[1+ \tfrac{1}{2}(y-d)^2 / D^2]$$

$$= D + (y-d)^2 /2D \qquad .... (4.13)$$

Using equation (4.12) and (4.13), the path difference becomes

$$\Delta = D + \frac{(y+d)^2}{2D} - D - \frac{(y-d)^2}{2D} = \frac{2yd}{D} \qquad .... (4.14)$$

For the position of bright fringes path difference

$$\Delta = n\lambda \qquad \text{(where n=1, 2, 3……..)}$$

or $$\frac{2yd}{D} = n\lambda$$

or $$y = \frac{nD\lambda}{2D}$$

Since the expression consists of integer *n*, i.e., *y* is a function of *n*. Thus it is better to use y_n in place of *y* and we can write,

$$y_n = \frac{nD\lambda}{2D} \qquad .... (4.15)$$

Where n = 1, 2 … etc. represents the order of fringe

On putting the value of n=1, n=2 etc. we get the bright fringes at positions $y_1 = \frac{D\lambda}{2D}$, $y_2 = \frac{2D\lambda}{2D}$ etc. Similarly for the position of dark fringes, the path difference should be

$$\Delta = \frac{(2n-1)\lambda}{2}$$

or $$\frac{2yd}{D} = \frac{(2n-1)\lambda}{2}$$

or $$y_n = \frac{(2n-1)}{2}\frac{D\lambda}{2D} \qquad .... (4.16)$$

If we place the value of n = 1, 2, 3 … we get the positions of dark fringes at $y_1 = \frac{1}{2}\frac{D\lambda}{2D}$, $y_2 = \frac{3}{2}\frac{D\lambda}{2D}$, $y_3 = \frac{5}{2}\frac{D\lambda}{2D}$…… etc.

Fringe Width: Distance between two consecutive bright or dark fringes is called fringe width denoted by ω (sometimes β). In case of bright fringes, fringe width

$$\omega = y_{n+1} - y_n = (n+1)\frac{D\lambda}{2D} - n\frac{D\lambda}{2D} = \frac{D\lambda}{2D}$$

Similarly, in case of dark fringes

$$\omega = y_{n+1} - y_n = \frac{2(n+1)-1}{2}\frac{D\lambda}{2D} - \frac{(2n-1)-1}{2}\frac{D\lambda}{2D} = \frac{D\lambda}{2D}$$

4.8 COHERENCE LENGTH AND COHERENCE TIME

In case of ordinary light source, light emission takes place when an atom leaves it excited state and come to ground state or lower energy state. The time period for the process of transition from an upper state to lower state is about 10^{-8} s only. Therefore an excited atom emits light wave for only 10^{-8} s and wave remains continuously harmonic for this period. After this period, the phase changes abruptly. But in a light source, there are innumerous numbers of atoms which participate in the emission of light. The emission of light by a single atom is shown in figure 4.7. After the contribution of a large number of atoms emitting light photon, a succession of wave trains emits from the light source.

Figure 4.7

4.8.1 Coherence Length

Coherence length is propagation distance over which a coherent wave maintains coherence. If the path of the interfering waves or path different is smaller than coherent length, the interference is sustainable and we observe distinct interference pattern.

4.8.2 Coherence Time

Coherent time τ_c is defined as the average time period during which the wave remains sinusoidal and after which the phase change abruptly.

4.8.3 Spatial Coherence

Spatial coherence describes the correlation between waves at different points on a plane perpendicular to the direction of propagation. More precisely, the spatial coherence is the cross-correlation between two points in a wave for all times. If a wave has only 1 value of amplitude over an infinite length, it is perfectly spatially coherent.

4.8.4 Temporal Coherence

Temporal coherent describes the correlation between two points in the direction of propagation. In other words, it characterizes how well a wave can interfere with itself at a different time as direction of propagation indicates time line. The delay over which the phase or amplitude wanders by a significant amount (and hence the correlation decreases by significant amount) is nothing but coherence time τ_c

4.9 CONDITIONS FOR SUSTAINABLE INTERFERENCE

As we studied the different aspects of interference it is clear that under which conditions interference can take place. But for strong interference or sustained interference some more condition may be summarized. The conditions are:

1. The interfering waves must have same frequencies. For this purpose we can select a single source.
2. The interfering waves must be coherent. To maintain the coherence, the path difference of two interfering waves must be less than coherence length.
3. As fringe width is given by $\omega = \frac{D\lambda}{2d}$. Thus to obtain reasonable fringe width the distance between source and screen D should be large and distance 2d between two sources should be small.
4. For good contrast we can prefer the interfering wave of same amplitude. If amplitude of two waves, a_1 and a_2 are same or nearly same than we observe distinct maxima and minima.
5. The back ground of screen should be dark.

4.10 INTERFERENCE DUE TO THIN SHEET

When a thin transparent sheet of mica of thickness t and refractive index μ is introduced in the path of one of the interfering beam of light, then entire fringe system is displaced. Suppose a thin sheet of mica of thickness t is place in the path of a light beam as shown in figure 4.8 then suppose the fringe system is displaced by a distance x.

If *t* is the time taken by light to travel distance S_1P, then

$$t = \frac{S_1P - t}{c} + \frac{t}{v}$$

where v is velocity of light in the thin sheet and c is the velocity of light in air.

$$t = \frac{S_1P - t}{c} + \frac{t}{c}\mu \qquad \because \mu = \frac{c}{v}$$

$$t = \frac{S_1P - t + \mu t}{c}$$

For light ray reaching to P from slit S_1, the path travelled in air is S_1P-t while in thin sheet is t, the optical path can be written as

$$= S_1P - t + \mu t = S_1P + (\mu - 1)t$$

Now path difference between two interfering says S_1P and S_2P at P is given as

$$\Delta = S_2P - S_1P = S_2P - [S_1P + (\mu - 1)t]$$

$= S_2P - S_1P-(\mu-1)t$

$= \frac{2yd}{D} - (\mu-1)\ t$ (Using equation 4.14)

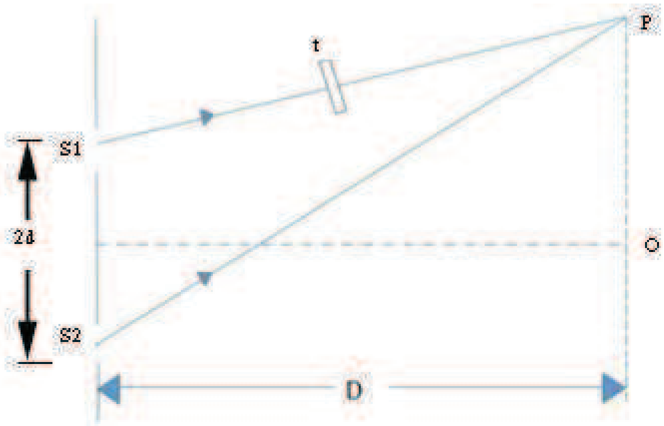


Figure 4.8

For nth maxima (bright fringe) path difference should be of the order of nλ, i.e.,

$$\frac{2yd}{\Delta} - (\mu-1)\ t= n\lambda$$

Taking y as y_n we get, $y_n= \frac{D}{2d}\ [n\lambda + (\mu-1)\ t]$ .... (4.17)

In the absence of thin sheet (t =0)

$$y_n = \frac{nD\lambda}{2d}$$

Therefore, net displacement in the presence and absence of sheet is given by equations 4.18 and 4.19 respectively

$$x= \frac{D}{2d}\ [n\lambda+ (\mu-1)\ t] - \frac{nD\lambda}{2d} \quad \text{....... (4.18)}$$

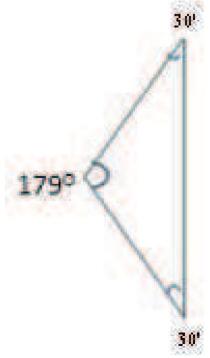
$$x = \frac{D}{2d}\ (\mu-1)\ t \quad \text{....... (4.19)}$$

Therefore, on introducing a thin transparent sheet in the path of any interfering ray, the entire fringe system will disposed by distance of x. By measuring the value of x we can calculate the thickness of sheet.

$$t = \frac{x\ .2d}{D(\mu-1)} \quad \text{....... (4.20)}$$

4.11 FRESNEL'S BIPRISM

Fresnel biprism consists of two acute angle prisms with their bases in contact. Generally the angles are 179^0, $30'$ and $30'$ as shown in figure 4.9. The light coming from a source is allowed to fall symmetrically on a biprism as shown in figure 4.9. As we know, when a light beam is incident on a prism, the light is deviated from its original path through an angle called angle of deviations. Similarly in case of biprism, the light beam coming from source S, is appeared to be coming from S_1 and S_2 as shown in figure 4.10. Thus we can say for prism S_1 and S_2 behave as virtual sources for the biprism.



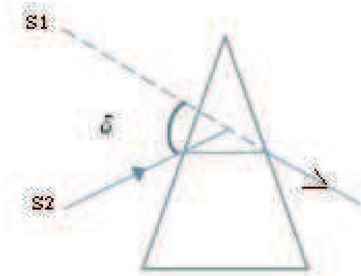


Fig. 4.9

In case of biprism, it can be considered that two cones of lights AS_1Q and BS_2P are coming from S_1 and S_2 and superimposed on each other and produce interference fringes in the region of superposition (between AB). The formation of interference fringes due to Fresnel's biprism is the same as due to Young's double slit experiment.

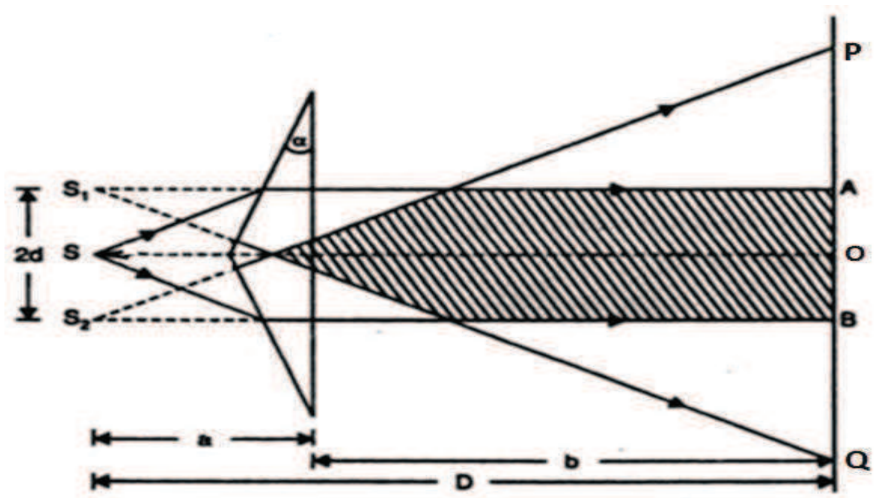


Fig. 4.10

In this experiment point O is equidistance from both slits S_1 and S_2. If we consider distance between source and screen is D and separation between two slits S_1 and S_2 is 2d the fringe width can be given as

$$\omega = \frac{D\lambda}{2d}$$

The position of n^{th} bright fringe is given by $y_n = n\frac{D\lambda}{2d}$

Similarly the position of n^{th} dark fringe is given by $y_n = \frac{2n-1}{2}.\frac{D\lambda}{2d}$

The wave length of the light source used in biprism experiment can be obtained by using above relation as

$$\lambda = \omega\frac{2d}{D} \qquad \text{....... (4.21)}$$

4.11.1 Experimental Arrangement of Biprism Apparatus

The experiment is performed on an optical bench as shown in figure 4.11. In this experiment we have an optical bench, which is an arrangement of two parallel metallic rods which are horizontal at same label. The rods or optical bench carry upright on which optical instruments are mounted. These upright are movable on the rods. In the first uprights, we have a slit illuminated by a monochromatic light source S. The slit provides a linear monochromatic light to the biprism which is mounted on the second upright. The biprism is placed in such a way that its refracting edges parallel to the slit so that light falls symmetrically on the biprism. In third upright there is a concave lens for conversing the light coming from biprism. Finally on forth upright a micrometer eyepiece is mounted in which interference fringes are observed.

For obtaining fringes, following adjustments are to be made.

(i) The optical bench is leveled with the help of spirit level.
(ii) Axis of slit is made parallel to edge of biprism.
(iii) The heights of all four uprights should be same so that line joining slit, biprism and micrometer should be parallel to optical bench.

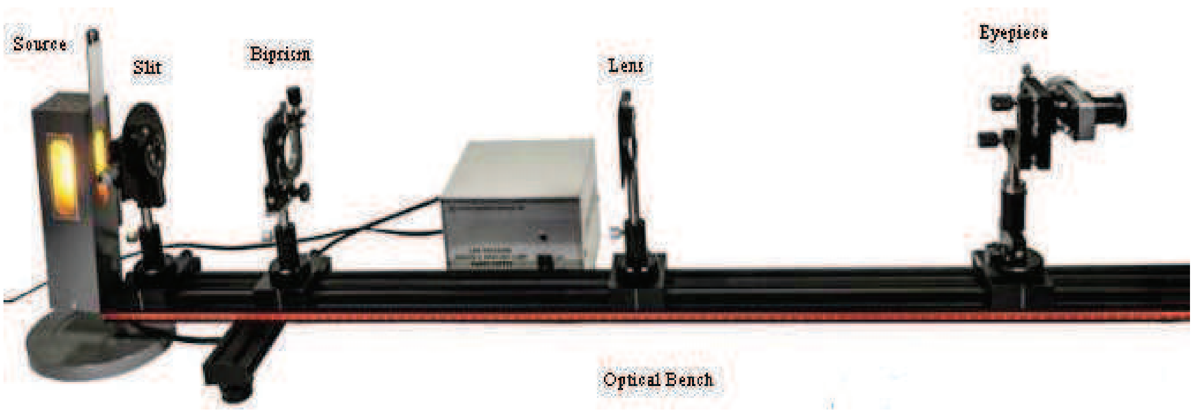


Figure 4.11

4.11.2 Lateral Shift

If the eyepiece of micrometer is moved away from the biprism, and fringes shift either left or right of bench then it is called lateral shift. Simply, we can say the shift of fringes across the bench is called lateral shift. It indicates that the line joining the slit biprism and eyepiece is not parallel to the optical bench.

To remove the lateral shift we put the eyepiece near the birprism and fix the vertical crosswire on any fringe. Now micrometer eyepiece is moved some distance away from biprism and direction of fringe shift is observed. Now biprism is moved in the direction opposite to the fringe shift so that vertical crosswise again reached on same fringe. We repeat this process again and again so that lateral shift removes compatibly.

4.11.3 Measurement of Wavelength of Light (λ) by Fresnel Biprism

By using the Fresnel biprism we can determine the wavelength of given source of light. For this purpose we use the given light source in experimental arrangement. We adjust the apparatus for fringes are to be observed on the eyepiece. We measure the fringe width on apparatus and apply the formula for fringe width as

$$\omega = \frac{D\lambda}{2d} \quad \text{or} \quad \lambda = \omega . \frac{2d}{D}$$

Fringe width ω can be measured with the help of micrometer on eyepiece. D is the distance between eyepiece and slit, and can be measured with the help of optical bend. The 2d is the distance between two virtual sources (S_1 and S_2) and cannot be measured directly with the help of any scale. We apply two methods for the measurement of distance 2d.

Magnification Method

To determine the distance 2d, we placed a convex lens of short focal length between biprism and screen. We find out a position L_1, of lens very near to biprism so that two sharp real images are obtained in the field of view of eyepiece. In figure 4.12 the position of Lens L_1 is denoted by bold lines. In this position, we measure distance between two images d_1, with the help of micrometer of eyepiece.

For this position the magnification is given by

$$\frac{v}{u} = \frac{d_1}{2d}$$

Now we move the lens some distance away from the biprism and obtain another position L_2 so that two sharp images are seen again in the field of view. We again measure the distance between two images, say d_2 with the help of micrometer of eyepiece.

In this case of position L_2 the magnification is given as

$$\frac{u}{v} = \frac{d_2}{2d}$$

By using above two equations (10) and (11) we get:

$$1 = \frac{d_1}{2d} . \frac{d_2}{2d}$$

or $$2d = \sqrt{d_1 d_2} \qquad .... (4.22)$$

By putting the value of d_1 and d_2 we can determine the value of 2d.

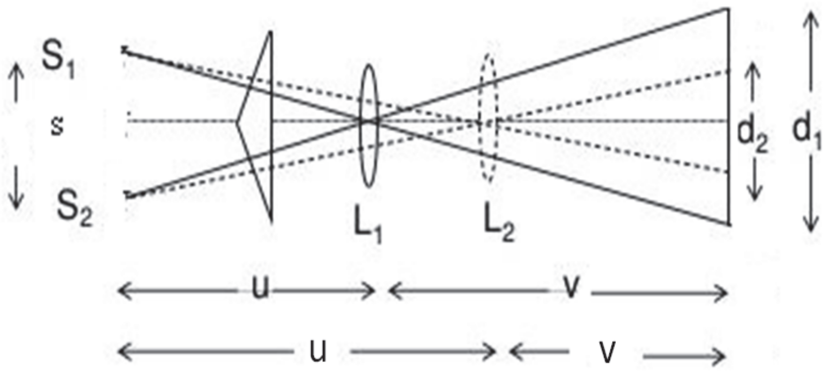


Figure 4.12

Refractive Index Method

In this method, we use the formula of angle of deviation for a prism. As shown in figure 4.13 the angle of deviation can be given as

$$\delta = (\mu - 1)\,\alpha \qquad .... (4.23)$$

Where μ is refractive index and α is angle of prism as shown in figure 4.13. Again the angle of deviation can be given as.

$$\delta = \frac{d}{a} \text{ or } d = a\,\delta \qquad .... (4.24)$$

Using equations (4.23) and (4.24), we obtain, 2d = 2a δ

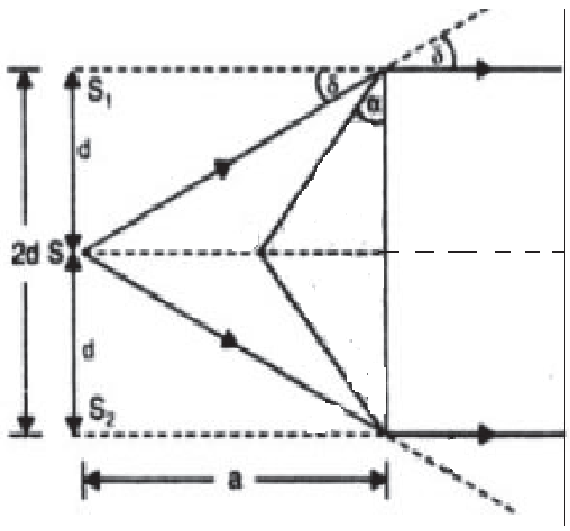


Figure 4.13

or $$2d=2a(\mu-1)\alpha \qquad .... (4.25)$$

By using any of the above mentioned methods, we can determine the value of 2d and then putting this value is equation 4.21, we can determine the wavelength of given light source.

4.12 INTERFERENCE WITH WHITE LIGHT

Now let us discuss what happen when the monochromatic light source in a Young's double slit experiment is replaced by a white light. Since the white light consists innumerable wavelengths from red to violet, when white light is used, all wavelengths have their own fringe pattern and finally superimposed on each other. Since the path different for all colours at center point is same then the waves of all colours reach at mid point without any path difference and we observed a white fringe at Center point. This central fringe is called zero order fringes. After central fringe, we observed few coloured fringes with poor contrast. These fringes are due to superposition of different fringes of different colours. Thus the interference pattern is not clear but the superposition of many colours.

Self Assessment Questions

1. What is difference between coherence and non coherence light?
2. Why non-coherent sources do not produce interference pattern?
3. What are the conditions for sustainable interference?
4. Young's double slit experiment, why the central fringe is bright?
5. How can we arrange coherence sources in practical?
6. What is meant by interference of light?
7. Explain the principle of superposition of light wave?
8. How is the shape of fringes formed by biprism?

4.13 SOLVED EXAMPLES

Example 4.2: A monochromatic light of wave length 5100 Å from a slit is incident on a double slit. If the overall separation of 30 fringes on a screen 200 cm always is 3cm, find the distance between slits.

Solution: The fringe width $\omega = \frac{D\lambda}{2D}$

Where ω = fringe width, D = distance between slit and screen, 2d= distance between slits.

It is given that D =200 cm, $\omega = \frac{3}{30} = 0.1cm$

Therefore, $2d = D\lambda/\omega = \frac{5100 \times 10^{-8} \times 200}{0.1} = 0.025$ cm

Example 4.3: In Young's double slit experiment the two slits are 0.05 mm apart and screen is located 2m away from the slit. The third bright fringe from the slit is displaced 8.3 cm apart from the central fringe. Determine the wavelength of incident light.

Solution: For the third bright fringe n= 3

$x_n = \frac{nD\lambda}{2d}$ or $\lambda = \frac{x_n . 2d}{nD} = \frac{8.3\times10^{-2}\times0.05\times10^{-3}}{3\times2} = 6.91 \times 10^{-7}$ m = 6910 Å

Example 4.4: In Fresnel's biprism experiment, a light of wavelength 6000 Å falls on biprism. The distance between source and screen is 1m and distance between source and birprism is 10 cm. The angle of biprism is 1^0. If the fringe width is 0.03cm, find out the refractive index of the material of biprism.

Solution: The fringe width $\omega = \frac{D\lambda}{2d}$

If the refractive index of material is μ and angle of prism is α then

$2d = 2a (\mu-1) \alpha$. Then $\omega = \frac{D\lambda}{2a\ \omega(\mu-1)\ \alpha}$

Here, D = 1m = a+b and a= 10 cm, b= 90cm, λ= 6000× 10^{-8} cm, $\alpha = 1^0 = \frac{\pi}{180}$ radian and ω = 0.03 cm

Thus, $\mu - 1 = \frac{D\lambda}{2a\ \omega\alpha} = \frac{100\times6000\times10^{-8}}{2\times10\times0.03\times\frac{\pi}{180}} = 0.57$

∴ $\mu = 1+0.57 = 1.57$

Example 4.5: A light of wavelength 6900 Å is incident on a biprism of refracting angle 1^0 and refractive index 1.5. Interference fringes are observed on a screen 80 cm away from the biprism. If the distance between source and the biprism is 20 cm, calculated the fringe width.

Solution : The fringe width is given by $\omega = \frac{D\lambda}{2d}$ and $2d = 2(\mu-1)a\alpha$

Here λ=6900 Å =6900 ×10^{-8} cm, $\alpha = 1^0 = \frac{\pi}{180}$ redius, μ = 1.5, D = a+b = (20+80) cm =100cm

$$\omega = \frac{D\lambda}{2a(\mu-1)\alpha} = \frac{100\times6900\times10^{-8}}{2\times20\times(1.5-1)\times\frac{\pi}{180}} = 0.02 \text{ cm.}$$

Example 4.6: A thin sheet of a transparent material of refractive index μ =1.60 is placed in the path of one of the interfering beam in a biprism experiment. The wave length of the light used is 5890Å. After placing the sheet, the central fringe shifted to a position originally occupied by 12^{th} bright fringe. Calculate the thickness of the sheet.

Solution: On introducing a thin transparent sheet in the path of one interfering say, the interfering system is shifted by a distance x and

$$x = \frac{D}{2d}(\mu - 1)t$$

In this case the fringe shifted by 12th bright fringe.

$$x = y_{12} = 12\frac{D}{2d} \qquad [\because y_n = n\frac{D\lambda}{2d}]$$

Therefore, $12\frac{D\lambda}{2d} = \frac{D}{2d}(\mu - 1)t$ or $t = \frac{12\lambda}{(\mu-1)} = \frac{12\times 5890\times 10^{-8}}{(1.6-1)} = 1.18 \times 10^{-3}$ cm

4.14 SUMMARY

1. When two light waves of same frequency and nearly some amplitude and having constant phase difference traverse in a medium and cross each other, there is redistribution in the intensity of light which is called interference of light.
2. If $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t+\delta)$ are two waves, then resultant wave is given by $y = A \sin (\omega t+\emptyset)$. Where $A=\sqrt{a_1^2 + a_2^2 + 2a_1a_2 Cos\delta}$ and $\emptyset = tan^{-1}\left[\frac{a_2 sin\delta}{a_1+a_2 cos\delta}\right]$
3. For constructive interference or bright fringes, path difference $\Delta = n\lambda$ where n=1, 2, 3…
4. For destructive interference or dark fringes, path difference $\Delta = (\frac{2n-1}{2})\, n\lambda$
5. For sustainable interference the two waves should be coherent. If two or more waves of same frequency are in the same phase or have constant phase difference then there waves are called coherent.
6. In interference pattern, the component of energy (intensity) $2a_1 a_2$ is simply transfer from minima to maxima point. The net intensity or average intensity remains constant or conserved.
7. Interference is of two types, known as division of wave front and division of amplitude.
8. Division of wave front is a class of interference in which the light from original common source is divided into two parts by employing mirror, prism, lens, biprism etc.
9. In case of division of amplitude, the incoming beam is divided into two or more parts by partial reflection or refraction. Interference due to thin film, Newton's rings, Michelson interferometer are the examples of division of amplitude.
10. In Young's double slit experiment fringe width is given by $\omega = \frac{D\lambda}{2d}$. Sometimes symbol β is to be used for fringe width. Position of n^{th} bright fringe is given by $y_n = n\frac{D\lambda}{2d}$. Similarly position of nth dark fringe $y_n = \frac{2n-1}{2}\frac{D\lambda}{2d}$, where D distance between slit and screen and 2d is the separation between slits S_1 and S_2.

11. On introducing a thin transparent sheet of thickness t in the path any interfering ray, the entire fringe system will be displaced by a distance x given as x = $\frac{D}{2d}$(μ-1) t.
Where μ is refractive index of material of sheet, 2d is distance between two slits.

12. In Fresnel's biprism the fringe width is given by ω = $\frac{D\lambda}{2d}$ and 2d = 2a (μ-1)α where a is distance between source and biprism and α is the angle of biprism and μ is refractive index of material of biprism.

4.15 GLOSSARY

Interference: Redistribution of energy due to superposition of waves.

Interference fringes: Pattern of dark and bright bands due to interference.

Superposition: Combining the displacements of two or more waves to produce a resultant displacement.

Coherence: Property of two or more waves with equal frequency and constant phase difference.

Coherent light: Light in which all wave trains have same frequency and its crests and troughs aligned in same directions which have constant phase difference.

Biprism: Combination of two prisms with their bases in contact.

Slit: A narrow opening for light.

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4.18 TERMINAL QUESTION

4.18.1 Short Answer Type Questions

1. What is interference of light? Give some example of interference of light.
2. What are the necessary conditions for interference of light?
3. What are coherent sources of light?
4. Discuss why two independent sources of some frequency are not coherent?
5. State the principle of superposition of waves.
6. Explain the optical path of light in a medium.
7. What is the difference between ordinary prism and biprism? How can we distinguish?

4.18.2 Long Answer Type Questions

1. What is interference of light? Obtain the condition for constructive and distractive interference.
2. What is Young's double slit experiment? Find out the position of bright fringes, dark fringes and fringe width.
3. Derive an expression for the resultant intensity of two coherent beam of light which are superimposed.
4. Explain the construction and working of biprism.
5. Calculate the displacement of fringe system when a transparent thin film is introduced in the path of an interfering beam in the double slit experiment.

4.18.3 Numerical Questions

1. A biprism is placed 5 cm from the slit and 75cm from the screen. The biprism is illuminated by sodium light of wavelength 5890Å. The fringe width is observed 424×10^{-2} cm. Calculate the distance between two coherent sources. [Ans. 0.5mm]
2. A biprism form interference fringes with monochromatic light of wave length 5450Å. On introducing a thin glass plate of refractive index 1.5 in the path of one of the interfering beam, the central fringe shifts to the position previously occupied by 6^{th} bright fringe. Find out the thickness of the plate.
3. The inclined faces of a biprism of refractive index 1.5 make angle 2^0 with base. A slit illuminated by a monochromatic light is placed at a distance of 10cm from the biprism. If the distance between two dark fringes observed at a distance of 1cm from the biprism is 0.18 mm, find out the wavelength of light used.

4. The inclined faces of a glass biprism of refractive index 1.5 makes angle of 1^0 width base of the prism. The distance between slit and biprism is 0.1m. The biprism is illuminated by a light of wavelength 5900Å and fringes are observed at a distance 1m from the biprism. find out the fringe width.

4.18.4 Objective Type Questions

1 . Phase difference Φ and path difference δ are related by Φ=

(a) $\frac{2\pi}{\lambda}\delta$ (b) $\frac{\lambda}{2\pi}\delta$

(c) $\frac{\pi}{2\lambda}\delta$ d) $\frac{2\lambda}{\pi}\delta$

2 . The condition for constructive interference is path difference should be equal to

(a) odd integral multiple of wavelength
(b) integral multiple of wavelength
(c) odd integral multiple of half wavelength
(d) Integral multiple of half wavelength

3. The ratio of intensities of two waves that produce interference pattern is 16:1 then the ratio of maximum and minimum intensities in the pattern is

(a) 25:9 (b) 9:25 (c) 1: 4 (d) 4:1

4. Correlation between the a point in the field and the same point in the field at later time is known as

(a) Temporal coherence (b) coherence

(c) Spatial coherence (d) none of these

5. The overlapping of waves into the regions of the geometrical shadow is

(a) Dispersion (b) polarization

(c) diffraction (d) interference

6. Interference occurs due to

(a) Wave nature of light (b) particle nature of light

(c) both a and b (d) none of these

7. Two interfering beams have their amplitudes ratio 2:1 then the intensity ratio of bright and dark fringes is

(a) 2:1 (b) 1:2 (c) 9:1 (d) 4:1

8. If a_1 and a_2 are the amplitudes of light coming from two slits in Young's double slit experiment then the minimum intensity of interference fringe is

(a) $a_1 + a_2$ (b) $a_1 - a_2$ (c) $(a_1 + a_2)^2$ (d) $(a_1 - a_2)^2$

9. Young's double slit experiment is an example of division of

(a) amplitude (b) Wavelength (c) wave front (d) None

10. In Young's double slit experiment, the fringe width ω is given by

(a) $\frac{D\lambda}{2d}$ (b) $\frac{D\lambda}{d}$ (c) $\frac{2d\lambda}{D}$ (d) $\frac{d\lambda}{2D}$

4.18.5 Answers of Objective Type Questions

1. (a), 2. (b), 3(a), 4(a), 5(d), 6(a), 7(c), 8(c), 9(c), 10(a)

UNIT 5: INTERFERENCE IN THIN FILMS AND NEWTON'S RINGS

CONTENTS

5.1. INTRODUCTION

In optics any transparent material in a shape of thin sheet of order 1μm to 10 μm is simply called thin film. The material may be glass, water, air, mica and any other material of different refractive index. When a thin film is illuminated by a light, some part of incident light get refracted from the upper surface of film and some part of get transmitted into the film. Some part of transmitted light gets reflected again from the lower surface of thin film. Now the light reflected from upper and lower surface of thin may course interference.

In case of thin film, the maximum portion of incident light is transmitted and a very few part of light reflected form the thin film. Therefore the intensity of reflected light is significantly small. For example if we consider a light beam is reflected from a glass plate of refractive index 1.5 then the reflection coefficient is given by

$$r = \left(\frac{\mu_1 - \mu_2}{\mu_1 - \mu_2}\right)^2 = \left(\frac{1.5-1}{1.5+1}\right)^2 = \left(\frac{0.5}{2.5}\right) = 0.04$$

Thus only 4% of incident light is reflected by the upper surface of glass film and 96% of light is transmitted into the glass plate. Similarly nearly 4% of light is again reflected through the lower surface of glass plate. If we consider the interference due to the light reflected from upper and lower surface of glass plate, the intensity of light will be significantly small.

When white light is incident of thin film, interference pattern is appeared as colourful bands since white light consists different wavelengths, different wavelengths produce interference bands of different colours and thicknesses. Interference in thin films also occurs in nature. Thin wings of many insects and butterflies are layer of thin films. There thin films are responsible for structural colourization which produce different colours by microscopically structured surface, and suitable enough for interference of light.

5.2. OBJECTIVE

After reading this unit you will be able to understand

- Thin film
- Interference in thin film
- Interference in wedge shaped film
- Classification of fringes and its shapes
- Newton's rings experiments and its applications

5.3. INTERFERENCE DUE TO PLANE PARALLEL THIN FILM

A plane parallel thin film is transparent film of uniform thickness with two parallel reflecting surfaces. The example is a thin glass film. Light wave generally suffers multiple reflections and refractions at the two surfaces. There are two cases of interference as given below

5.3.1 Interference in Case of Reflected Light

Let us consider a thin film of thickness *t* as shown in figure 5.1. A monochromatic light ray SA is incident on a thin film with an angle of incident *i* as shown in figure. The film is made of a transparent material (say glass) of refractive index μ. Some part of light ray reflected at point A along the direction AB and some part of light transmitted into the film along AC direction. The ray AC makes an angle of refraction *r* at point A, and the angle r becomes angle of incident ACN at point C. Some part of light of ray AC again reflected in the direction CD which comes out from the film along the direction DE. The light rays AB and DE come together and they can produced interference pattern on superposition.

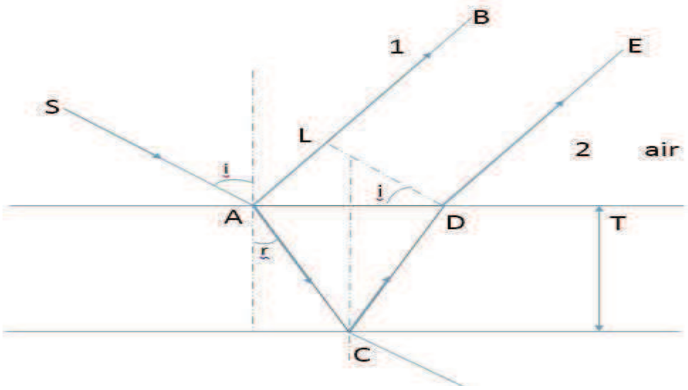


Fig 5.1

The path difference Δ between rays AB and DE is given as

$$\Delta = (AC+DC) \text{ in film- } AL \text{ in air.}$$

Since optical path in air = μ × optical path in a medium

Therefore, path difference Δ can be given as

$$\Delta = \mu\,(AC+ DC) - AL$$

From figure 5.1, we have, $\cos r = \frac{t}{AC}$ or $AC = \frac{t}{\cos r}$ and $DC = \frac{t}{\cos r}$

Again, $AL = AD \sin i = (AN+ ND) \sin i$

$$= (t \tan r + t \tan r) \sin i = 2t \tan r \, Sin\, i$$

$$\Delta = \frac{\mu\, 2t}{\cos r} - 2t.\tan r \sin i = \frac{2\mu t}{\cos r} - 2\mu t\,(\sin^2 r)$$

$$= 2\mu \frac{t}{Cos\, r}(1 - \sin^2 r) = 2\mu t \cos r$$

According to Stock's treatment, if a wave is reflected form a denser medium it involves a path difference of λ/2 or phase difference or π. Therefore, net path difference

$$\Delta = 2\mu t\, Cos\, r - \frac{\lambda}{2} \qquad \text{....... (5.1)}$$

Condition of Maxima: For maxima or bright fringes the path difference should be $n\lambda$ where n is integer number given as n= 0,1,2,3 ……..

$$\Delta = 2\mu t\, Cos\, r - \frac{\lambda}{2} = n\lambda$$

or $$2\mu t \, Cos\, r = (\frac{2n+1}{2})\lambda \qquad .... (5.2)$$

Thus maxima occur when optical path difference is $(\frac{2n+1}{2})\lambda$.

Condition for minima: Minima occur when the path difference is order of $(\frac{2n-1}{2})\lambda$. Then

$$\Delta = 2\mu t \, Cos\, r - \frac{\lambda}{2} = (\frac{2n-1}{2})\lambda$$

or $$2\mu t \, Cos\, r = n\lambda \qquad .... (5.3)$$

5.3.2 Interference in Case of Refracted Light

A light ray SA is incident at point A on a film of refractive index μ as shown in figure 5.2. Some part of light ray reflected at point A and some part of light transmitted into the film along AB. In case of interference due to refracted light we are not interested in the reflected light. At point B some part of light is again reflected along direction BC, then again reflected at point C and finally refracted at point D and comes out form the medium along DF direction. Now the light rays coming along BE and DF are coherent and can produce interference pattern in the region of superposition.

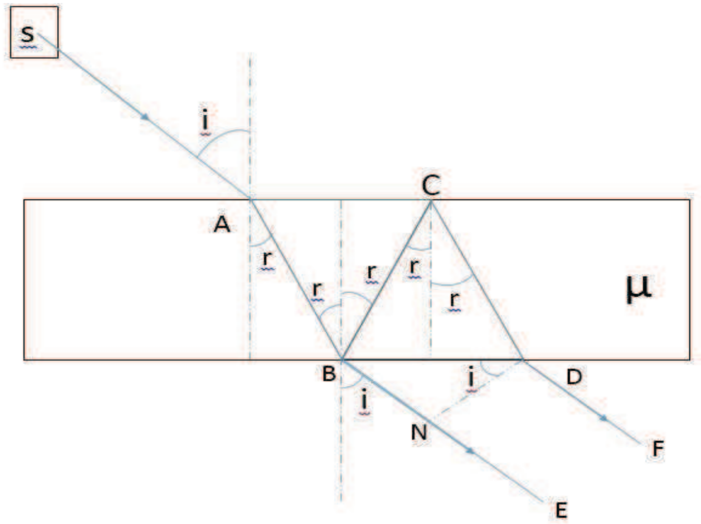


Figure 5.2

In this case path difference Δ is given as

$$\Delta = (BC + CD) \text{ in film} - BN \text{ in air}$$

As Calculated in case of reflection, the path difference comes out

$$\Delta = 2\mu t \, Cos\, r$$

In this case there is no correction according to Stoke's treatment as no wave from rarer medium is reflected back to denser medium. Therefore this is net path difference.

For maxima or bright fringes, $\Delta = 2\mu t \, Cos\, r = n\lambda$

For minima or dark fringes, $\Delta = 2\mu t \, Cos\, r = (\frac{2n-1}{2})\lambda$

5.4 *INTERFERENCE IN A WEDGE SHAPED FILM*

In a wedge shape film, the thickness of the film at one end is zero and it increases consistently towards another end. A glass wedge shaped film is shown in figure 5.3. Similarly a wedge shaped air film can be formed by using two glass films touch at one end and separated by a thin wire at another end.

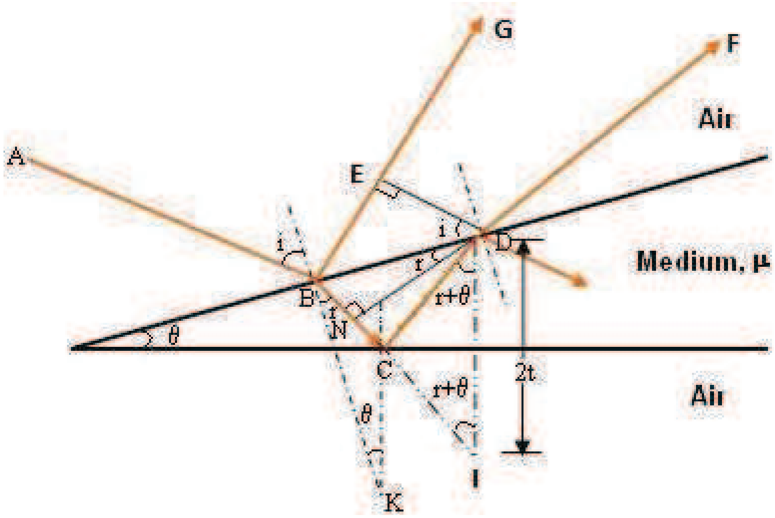


Figure 5.3

The angle made by two surfaced at touching end of wedge is called angle of wedge as shown ϴ in figure 5.3. The angle is very small in order of less than 1^0. Path difference between two reflected rays BE and DF is given by

$$\Delta = (BC+CD) \text{ in film} - BE \text{ in air}$$

$$= \mu\,(BC+CD) - BE$$

$$=\mu\,(BC+CI) - BE \qquad \because CD = CI$$

$$= \mu\,(BN+NI) - BE \qquad \text{.......}\ (5.4)$$

In right triangle Δ BED, $\sin i = \frac{BE}{BD}$

Similarly in Δ BND, $\sin r = \frac{BN}{BD}$

Refractive index μ can be given as

$$\mu = \frac{Sin\ i}{Sin\ r} = \frac{BE}{BN} \quad \text{or} \quad BE = \mu\, BN$$

Putting this value in equation (5.4) we get

$$\Delta = \mu\,(BN + NI) - \mu\, BN = \mu\, NI \qquad \text{.......}\ (5.5)$$

Now in Δ DNI, $\cos(r+\theta) = \frac{NI}{DI}$

or $cos\ (r + \Theta) = \frac{NI}{2t} \Rightarrow NI = 2t\ Cos\ (r + \Theta)$

Putting this value in equation (5.5)

Path difference, $\Delta = \mu.\ 2t\ Cos\ (r + \Theta)$ .... (5.6)

Since the light is reflecting from a denser medium therefore according to stokes treatment a path change of λ/2 occurs. Now net path difference

$$\Delta = 2t\ Cos\ (r + \Theta) - \lambda/2 \quad .... (5.7)$$

For bright fringes the path difference should be in order of Δ = nλ where *n* is an integer (n= 0, 1, 2…....).

$$2\mu t\ Cos\ (r + \Theta) - \lambda/2 = n\lambda$$

or $$2\mu t\ Cos\ (r + \Theta) = \left(\frac{2n+1}{2}\right)\lambda \quad \text{where } n = 0,1,2.....$$

or $$2\mu t\ Cos\ (r + \Theta) = \left(\frac{2n-1}{2}\right)\lambda \quad .... (5.8)$$

Where, *n* = 1, 2, 3…..

For dark fringes path difference should be in order of $\Delta = \left(\frac{2n-1}{2}\right)\lambda$.

$$2\mu t\ Cos\ (r + \Theta) - \lambda/2 = \left(\frac{2n-1}{2}\right)\lambda$$

or $$2\mu t\ Cos\ (r + \Theta) = n\lambda \quad .... (5.9)$$

Since the focus of points of constant thickness is straight line, therefore the fringes are straight lined in shape.

According to equation (5.8), for bright fringes

$$t = \frac{(2n-1)\lambda}{4\ \mu \cos(r+\theta)} = \frac{\lambda}{4\ \mu \cos(r+\theta)} = \frac{3\lambda}{4\ \mu \cos(r+\theta)} = \cdots \ldots \ldots \quad .... (5.10)$$

If x_n is the distance of fringes from the edge (position of n[th] fringe) then,

$$\tan\theta = \frac{t}{x_n}$$

or $$x_n = \frac{(2n-1)\lambda}{4\ \mu \cos(r+\theta) \tan\theta} \quad .... (5.11)$$

Thus, $$x_1 = \frac{\lambda}{4\ \mu \cos(r+\theta) \tan\theta},\ x_2 = \frac{3\lambda}{4\ \mu \cos(r+\theta) \tan\theta} \ldots\ldots\ldots\ldots$$

Fringe width $\omega = x_{n+1} - x_n$

$$\omega = \frac{2\lambda}{4\ \mu\ Cos(r+\theta) \tan\theta}) \quad .... (5.12)$$

If Θ is very small then tan Θ≃ Θ, and cos (r + Θ) ≅ r. Further if we consider normal incidence then r= 0^0 then cos 0 = 1 and equation (5.12) becomes

$$\omega = \frac{\lambda}{2\ \mu\ \theta} \quad .... (5.13)$$

5.4.1 Properties of Fringes Due to Wedge Shaped Film

1. As the locus of the points of constant thickness is a straight line therefore the fringes are straight lime and parallel.

2. The fringe width ω is constant for a particular wave length or colour, therefore the fringes are of equal thickness and equidistant.

3. Fringes are localized

5.4.2. Applications of Wedge Shaped Film

By observing the interference pattern, the thickness of a spacer or wire which is placed between two films at one end can be determined. Suppose *t* is the thickness of a wire or spaces and *l* is length of wedge shaped film as shown in figure 5.4 then we can calculate the thickness of spacer as

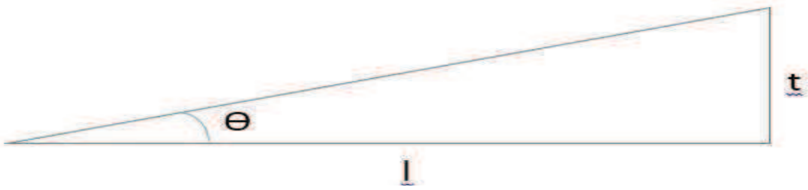


Figure 5.4

$$\tan \Theta \cong \Theta = \frac{t}{l}$$

If we know the fringe width ω then by using relation $\omega = \frac{\lambda}{2\mu\theta} = \frac{\lambda}{2\mu\frac{t}{l}}$ we get,

$$t = \frac{\lambda l}{2\mu.\omega}$$

Example 5.1: A white light is normally incident on a soap bobble film of thickness 0.40 μm and refractive index 1.4. Which are the wavelengths may cause bright fringes.

Solution: For bright fringes, due to thin films, the condition is

$$2\mu t \text{ Cos } r = (2n+1)\frac{\lambda}{2}, \text{ where } n= 0,1,2,3\ldots..$$

or $$\lambda = \frac{4\ \mu t \cos r}{(2n+1)}$$

Here r = 0, μ = 1.4 and t = 0.40 μm.

$$\lambda = \frac{4\times1.4\times0.40\times10^{-6}}{(2n+1)} = \frac{2.24\times10^{-6}}{(2n+1)}\ m$$

For n = 0; $\lambda = 2.24 \times 10^{-6}\ m$

n = 1; $\lambda = 0.74 \times 10^{-6}\ m$

n = 2; $\lambda = 0.44 \times 10^{-6}\ m$

Example 5.2: White light is incident on an oil film of thickness 0.01mm and reflected at an angle 45^0 to vertical. The refractive index of oil is 1.4 and refracted light falls on the slit of a spectrometer, calculate the number of dark bands seen between wavelengths 4000Å and 5000Å.

Solution: For the dark band, formed by interference, due to thin film

$$2\mu t \cos r = n\lambda$$

In case of wave length λ_1 =4000Å and μ=1.4 , t= 0.01 mm

$$n_1 = \frac{2\mu t \cos r}{\lambda_1}$$

Now $$\mu = \frac{\sin i}{\sin r} = \sin r = \frac{\sin i}{\mu}$$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{\sin^2 r}{\mu^2}} = \sqrt{1 - \frac{1}{2\times(1.4)^2}} = 0.86$$

Thus $$n_1 = \frac{2\times1.4\times0.001\times.86}{4000\times 10^{-8}} = 60$$

Thus corresponding to λ_1=4000Å wavelength light we observe 60^{th} order band

Similarly corresponding to λ_2 wavelength

$$n_2 = \frac{2\mu t \cos r}{\lambda_2} = \frac{2\times1.4\times.001\times0.86}{5000\times10^{-8}} = 48$$

Thus corresponding to wavelength λ_2 =5000Å light we observe 48^{th} order band.

Thus the number of dark bands between λ_2 and λ_1 = $n_1 - n_2$ =60-48 =12.

Example 5.3: A parallel beam of light λ = 5890 Å is incident on a thin glass film and the angle of refraction into the film is 60^0. Calculate the smallest thickness of the film which appear dark on reflection.

Solution: The film appears dark if the destructive interference takes place in reflection.

Path difference in dark bands

$$\Delta = 2\mu t \cos r = n\lambda$$

For smallest thickness n=01 then

$$t = \frac{\lambda}{2\mu \cos r} = \frac{5890\times10^{-10}}{2\times1.5\times0.5} = 3927 \times 10^{-10} m = 3927 \text{ Å}$$

Example 5.4: A monochromatic light of wavelength 5890 Å is incident normally on glass plates enclosing a wedge shaped air film. The two plates touch at one end and are separated at 15cm apart from that end by a wire of 0.05 mm diameter. Calculate the fringe width of bright fringes.

Solution: In case of wedge shaped film the fringe width is given by

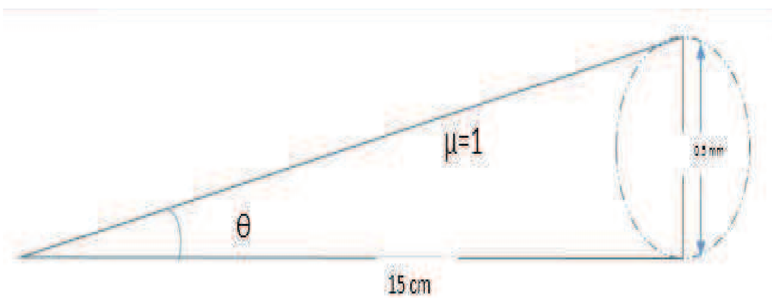


Figure 5.5

$$\omega = \frac{\lambda}{2\mu\theta}$$

Given λ = 5890 Å, μ=1, Θ= tan Θ = $\frac{0.05 \times 10^{-1}}{15} = 3.3\times 10^{-4}$

$$\omega = \frac{5890 \times 10^{-10}}{2\times 1.0\times 3.3\times 10^{-4}} = 892.4 \times 10^{-6} m = 0.89 \text{ mm}$$

Example 5.5: Sodium light of wavelength λ = 5890Å is incident on a wedge shaped air film. When viewed normally 10 fringes are observed in a distance of 1cm. Calculate the angle of the wedge.

Solution: The fringe width ω for wedge shaped film is given us

$$\omega = \frac{\lambda}{2\mu\theta}$$

In this case, 10 fringes are observed in a distance of 1 cm. Therefore, fringe width

$$\omega = \frac{1}{10} = 0.1\ cm$$

Now $\Theta = \frac{\lambda}{2\mu\omega} = \frac{5890 \times 10^{-8}}{2\times 2\times 0.1} = 2.94 \times\ 10^{-4}\ radians$

$= 2.94 \times\ 10^{-4} \times\ \frac{180}{л}$ degree $=3.94 \times\ 10^{-6} \times\ \frac{180\times 60}{л}$ minute

= 1.01 minute

Example 5.6: A Wedge shaped film is form by using two glass plates of length 10cm touch at one end and separate at another end by introducing a thin foil of thickness 0.02mm. If the sodium light of wavelength 5890Å is indent normally on it. Find the separation between two consecutive fringes.

Solution: The separation between two consecutive fringes is the same as the fringe width.

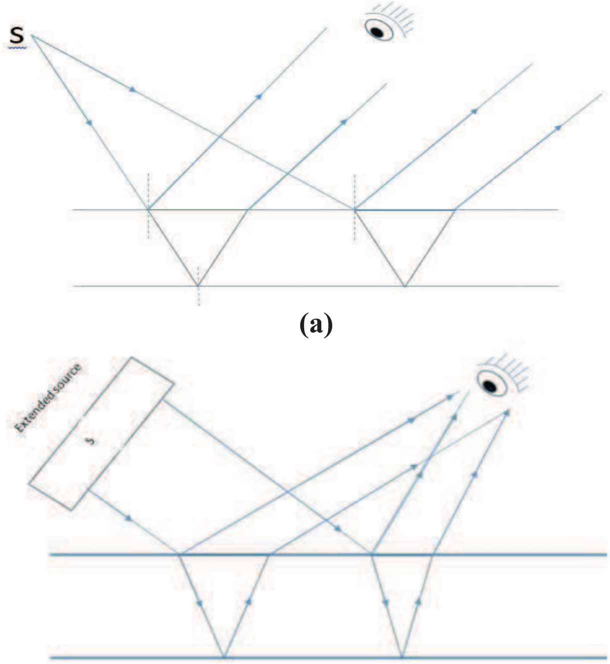
$\omega = \frac{\lambda}{2\mu\theta}$ where Θ = tan Θ = $\frac{t}{x} = \frac{0.02}{100} = 2 \times 10^{-4}$

Given λ =5890Å, μ = 1 then

$$\omega = \frac{5890 \times 10^{-8}}{2\times 1\times 2\times 10^{-4}} \text{ cm} = 0.14 \text{ cm}$$

5.5 NECESSITY OF EXTENDED SOURCE FOR INTERFERENCE DUE TO THIN FILMS

If we use narrow source of light in case of interference due to thin film the light rays are diverged as shown in figure 5.6 (a) and we can view a limited portion of interference pattern. On the other hand, if we use an extended or broad source of light a large number of rays are available for the production of interference pattern as shown in figure 5.6 (b). A large number of rays are incident on film at different angles, and a large area of film can be viewed by our eye at the field of view. Therefore, extended source of light is beneficial to observe the good interference pattern in thin film.



(b)

Figure 5.6

5.6 COLOURS OF THIN FILMS

When light coming from extended source is reflected by thin film of oil, mica, soap or coating etc., different colours are shown due to interference of light. For interference, the optical path difference is $\Delta = 2\mu t \, Cos \, r = (2n+1)\, \lambda/2$ for bright fringes. If thickness t is constant then for different wavelengths, angle of refraction r should be different. Therefore different colours are observed at different angle of incident. Sometime different colours are over lopped on each other's and a mixed colour may be observed.

5.7 CLASSIFICATION OF FRINGES

As we know, in case of thin films, the path difference Δ is given as

$$2\mu t \, Cos \, r = \left(\frac{2n+1}{2}\right)\lambda$$

For a monochromatic light, μ and λ remain constant. Now the path difference for constructive interference arises due to variation in thickness t and angle of incident (inclination) r. On the basis of t and r the fringes are two types.

5.7.1 Fringes of Equal Thickness

If the thickness of film is varying and the light is coming at same angle of incident then the fringes are formed due to variation in thickness. For example in case of wedge shaped film where thickness is varying, the locus of points of constant thickness is a straight line corresponding to which fringes are formed. Such fringes are called fringes of equal thickness. Newton's rings are example of such type of fringes.

5.7.2 Fringes of Equal Inclination

If the thickness of film is constant then path difference for constrictive interference is only due to variation in angle of inclination r. In this case we consider a locus of points on film at which the angle of inclination of light is equal. Corresponding to such points of equal inclination we observed fringes which are called fringes of equal inclination. Since the light rays of equal inclinations pass through the plate is a parallel beam of light, and hence meet at infinity but by using telescope focused on such rays the fringes can be observed. In such case fringes are called the fringes localized at infinity. Such fringes are also called Haidinger's fringes. The fringes formed in Michelson interferometer is an example of fringes of equal inclination.

5.8 NEWTON'S RINGS

Newton's rings in a special case of wedge shaped film in which an air film is formed between a glass plate and a convex surface of lens. The thickness of air film is zero at the center and increases gradually towards the outside.

When a plano-convex lens of large focal length is placed on a plane glass plate, a thin air film is formed between the lower surface of plano-convex lens and upper surface of glass plate. When a monochromatic light falls on this film the light reflected from upper and lower surfaces of air film, and after interference of these rays, we get an inner dark spot surrounded by alternate bright and dark rings called Newton's rings. These rings are first observed by Newton and hence called Newton's rings.

5.8.1 Experimental Arrangement for Reflected Light

The experimental arrangement for Newton's rings experiment is shown in Figure 5.7. A beam of light from a monochromatic source S is made parallel by using a convex lens L. The parallel beam of light falls on a partially polished glass plate inclined at an angle of 45^0. The light falls on glass plate is partially reflected and partially transmitted. The reflected light normally falls on the plano-convex lens placed on plane glass plate.

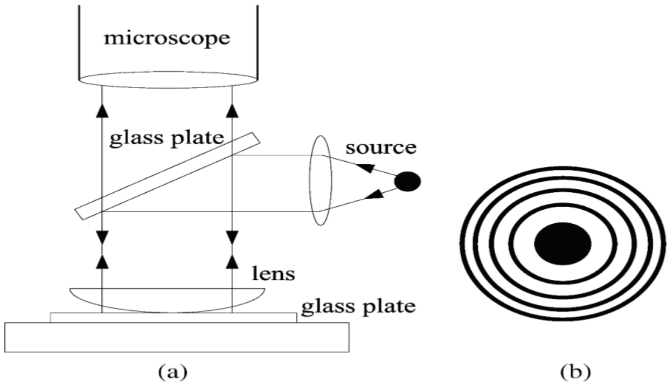


Fig. 5.7

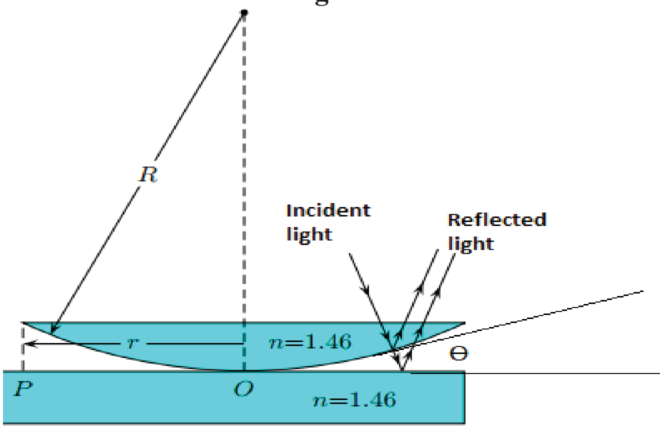


Fig 5.8

This light reflected from upper and lower surface of the air film form between plane glass plate and plno-convex lens. These rays interfere and rings are observed in the field of view. The figure 5.8 shows the reflection of light form upper and lower surfaces of air film which are responsible for interference.

5.8.2 Formation of Bright and Dark Rings

As we know the interference occurs due to light reflected from upper and lower surface of air film form between glass plate and plano-convex lens. The air film can be considered as a special case of wedge shaped film. In this case, angle wedge is the angle made between the plan glass plate and tangent from line of contact to curved surface of plano convex lens as shown in figure. 5.8.

The path difference between two interfering rays reflected by air film

$$\Delta = 2\mu t \text{ Cos } (r + \Theta) - \frac{\lambda}{2} \qquad \text{....... (5.14)}$$

where μ is the refractive index of the air film, t is the thickness of air film at the point of reflection (say point P) r is angle of refraction and Θ is angle of wedge.

In this case the light normally falls on the plane convex lens for the angle of refraction r = 0. Further, as we use a lens of large focal length the angle of wedge Θ is very small. So Cos (r+Θ) = Cos Θ = Cos 0^0 =1 and thus the path difference

$$\Delta = 2\mu t - \frac{\lambda}{2} \qquad \text{....... (5.15)}$$

At point of contact t = 0, therefore, $\Delta = \frac{\lambda}{2}$

Which is the condition of minima. Hence at centre or at point of contact there is a dark spot.

Condition of Bright Rings or Maxima

The condition for bright rings is path difference $\Delta = n\lambda$ therefore

$$\Delta = 2\mu t - \frac{\lambda}{2} = n\lambda \text{ where } n = 0, 1, 2, 3\ldots\ldots$$

or $$2\mu t = \left(\frac{2n+1}{2}\right)\lambda$$

or $$2\mu t = \left(\frac{2n-1}{2}\right)\lambda \qquad \ldots\ldots\ (5.16)$$

Where n = 1, 2, 3………

Condition of Dark Ring or Minima

In case of dark rings, the path difference, $\Delta = \left(\frac{2n-1}{2}\right)\lambda$

Where n= 1, 2, 3………

Therefore $$\Delta = 2\mu t - \frac{\lambda}{2} = \left(\frac{2n-1}{2}\right)\lambda$$

or $$2\mu t = n\lambda \qquad \ldots\ldots\ (5.17)$$

Thus corresponding to n = 1, 2, 3….. we observe first, second third…..etc. bright or dark rings. In Newton's rings experiment the locus of points of constant thickness is a circle therefore the fringes are circular rings.

5.8.3 Diameter of Bright and Dark Rings

In figure 5.9 the plano-convex lens BOPF is place on glass plate G and O is the point of contact. Suppose, C is the centre of the sphere OBFP from which the plano-convex lens is constructed. P is point on the air film at which the thickness of air film is t. At point P, the light is incident and reflected form the upper and lower surface of air film, and rings are formed. AP is the radius of ring passes through point P. According to property of circle

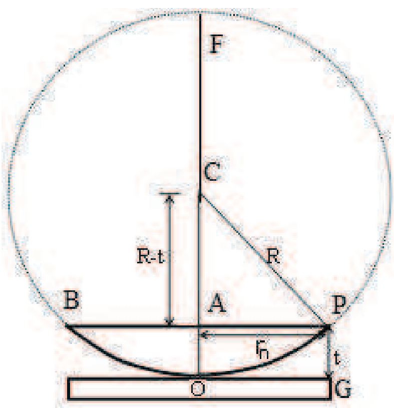


Figure 5.9

$$AP \times AB = AO \times AL$$

$$r^2 = t \times (2R - t) \quad \because AL=OL-OA$$

Where R is the radius of curvature of lens.

$$r^2 = 2Rt - t^2$$

Since R is very large and t is very small, we can write

$$r^2 = 2Rt \qquad \text{or} \qquad t = \frac{r^2}{2R}$$

Substituting this value of t in equation (5.16), we get,

$$2\mu \frac{r^2}{2R} = \left(\frac{2n-1}{2}\right)\lambda$$

or

$$r^2 = \left(\frac{2n-1}{2}\right)\frac{\lambda R}{\mu}$$

This expression contains n, *i.e.,* r is a function of n. Thus it is better to use r_n in place of r. If D_n is the diameter of nth bright ring then we have $r = r_n = D_n / 2$ and can write

$$\frac{{D_n}^2}{4} = \frac{\left(\frac{2n-1}{2}\right)\lambda R}{\mu}$$

or

$$D_n^{\ 2} = \frac{2(2n-1)\lambda R}{\mu} \qquad \text{....... (5.18)}$$

Where n= 1, 2, 3……. Similarly for dark rings

$$2\mu t = n\lambda \qquad \text{or} \qquad 2\mu \frac{r^2}{2R} = n\lambda \qquad \text{or} \qquad r^2 = \frac{n\lambda R}{\mu}.$$

If D_n is diameter of nth dark ring then

$$\frac{{D_n}^2}{4} = \frac{n\lambda R}{\mu}$$

or

$$D_n^{\ 2} = \frac{4n\lambda R}{\mu} \qquad \text{....... (5.19)}$$

Where n = 1, 2, 3……

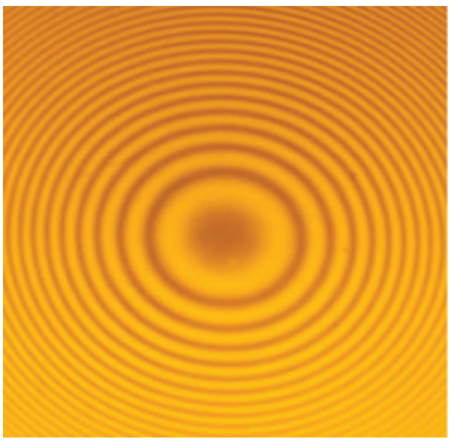


Figure 5.10

The alternate bright and dark rings are formed as shown in figure 5.10. The spacing between two consecutive rings can be given as

$r_{n+1}^2 - r_n^2 = \left(\sqrt{n+1} - \sqrt{n}\right) \lambda R$ (in case of air film μ =1)

Spacing between 1st and 2nd rings = $\left(\sqrt{2} - \sqrt{1}\right) \lambda R$ = 0.4142 λR

Spacing between 2nd and 3rd rings = $\left(\sqrt{3} - \sqrt{2}\right) \lambda R$ = 0.3178 λR

Spacing between 4th and 3rd rings = $\left(\sqrt{4} - \sqrt{3}\right) \lambda R$ = 0.21 λR

Thus it is clear that the spacing between successive rings decreases with increase in order.

5.8.4 Determination of Wave Length of a Monochromatic Light Source

In Newton's experiment if we use a light source of unknown wave length (say sodium lamp) then we can determine the wavelength of light source by measuring the diameters of Newton's ring.

If D_n is diameter of nth dark ring formed due to air film then

$$D_n^2 = 4n\lambda R$$

Where n is any integer number.

Similarly if $D_{(n+p)}$ is the diameter of $(n+p)^{th}$ ring

$$D_{n+p}^2 = \mu\,(n+p)\,\lambda R$$

Using this equation, we can write

$$D_{n+p}^2 - D_n^2 = 4\,(n+p)\,\lambda R - 4n\lambda R = 4\,P\,\lambda R$$

or $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \quad .... \quad (5.20)$$

Where p is any integer number and R is radius of curvature of plano-convex lens.

5.8.5. Determination of Refractive Index of a Liquid by Newton's Rings Experiment

In Newton's rings experiment the diameter of n^{th} dark ring in case air film is

$$D_n^2 = 4n\lambda R \qquad (\because \mu = 1)$$

The diameter of $(n+p)^{th}$ ring

$$D^2_{n+p} = 4(n+p)\lambda R$$

If a liquid of refractive index μ is filled between the plane glass plate and convex lens then

$$D^2_n = \frac{4n\lambda R}{\mu} \quad \text{and} \quad D^2_{n+p} = \frac{4(n+p)\lambda R}{\mu}$$

Thus we can write

$$\frac{[D_{n+p}^2 - D_n^2]air}{[D_{n+p}^2 - D_n^2]liquid} = \frac{4p\,\lambda R}{\frac{4p\,\lambda R}{\mu}} = \mu$$

or $$\mu = \frac{[D_{n+p}^2 - D_n^2]air}{[D_{n+p}^2 - D_n^2]liquid} \qquad .... (5.21)$$

Example 5.7: In Newton's rings experiment if the radius of curvature of plano-convex lens in 200 cm and wavelength of the light used is 5890 Å, calculate the diameter of 10^{th} bright ring.

Solution: The diameter of n^{th} bright ring is given as (μ=1 for air film) is given by

$$D_n^2 = 2(2n-1)\,\lambda\, R$$

or $$D_{10}^2 = 2 \times (20-1) \times 5890 \times 10^{-8} \times 200\ cm^2 = 6.69mm$$

The diameter of 10^{th} bright ring is 6.69 mm.

Example 5.8: In a Newton's ring experiment the diameter of 15^{th} dark ring and 5^{th} dark ring are 0.59 cm and 0.33cm respectively. If the radius of curvature of the convex lens is 100cm calculate the wave length of light used.

Solution: The wave length of unknown light source is Newton's rings experiment is given as

$$\lambda = \frac{[D_{n+p}^2 - D_n^2]}{4pR}$$

Here $D_{n+p} = D_{15} = 0.59$ cm, $D_n = D_5 = 0.33$cm, $p = 10$, R = 100cm

$$\lambda = \frac{(0.59)^2 - (0.33)^2}{4 \times 10 \times 100} = 5980\ Å$$

Example 5.9: Newton's rings are formed by using a monochromatic light of 6000Å. When a liquid is introduced between the convex lens and plane glass plate the diameter of 6^{th} bright ring becomes 3.1mm. If the radius of curvature of lens is 1mt, calculate the refractive index of liquid.

Solution: Given that, n= 6, $D_n = 3.1mm = 3.1 \times 10^{-3}$ m, $\lambda = 6000Å = 6 \times 10^{-7}$ m, R = 1m

$$\mu = \frac{2(2n-1)\lambda R}{D_n^2} = \frac{2 \times 11 \times 6 \times 10^{-7} \times 1}{(3.1 \times 10^{-3})^2} = 1.37$$

Example 5.10: In Newton's ring experiment two light sources of wavelength 6000Å and 4500Å are used to form rings. It is observed that n^{th} dark ring due to 6000Å light coinside with $(n+1)^{th}$ dark ring due to 4500Å. If the radius of curvature of the plano convex lens is 100cm, calculate the diameter of n^{th} dark ring due to λ_1 and λ_2.

Solution: For n^{th} dark ring due to λ_1, $D^2_n = 4n\,\lambda_1 R$

Similarly for $(n+1)^{th}$ dark ring due to λ_2, $D^2_{n+1} = 4\,(n+1)\,\lambda_2 R$

Since n^{th} dark ring due to λ_1 co-inside with $(n+1)^{th}$ dark ring due to λ_2 therefore.

$4n\lambda_1 R = 4(n+1)\lambda_2 R$ or $n\lambda_1 = (n+1)\lambda_2$ or $n\lambda_1 - n\lambda_2 = \lambda_2$ or $n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$

Here $\lambda_1 = 6000Å$, $\lambda_2 = 4500Å$

∴ $$n = \frac{4500}{6000-4500} = 3$$

Now the diameter of n=3[rd] dark ring due to λ_1

$$D_3^2 = 4n\lambda_1 R = 4\times3\times6000\times10^{-10}\times1 \text{ m}$$

Or $$D_3 = 2.68 \text{ mm.}$$

Similarly diameter of n=3[rd] dark ring due to λ_2

$$D_3^2 = 4n\lambda_2 R = 4\times3\times4500\times10^{-10}\times1 \text{ m}$$

or $$D_3 = 2.32 \text{ mm.}$$

Same relation can also be obtained for bright rings.

5.8.6 Newton's Rings in Case of Transmitted Light

The Newton's rings can also be formed in case of interference due to transmitted light as shown in figure 5.11. In this case the transmitted rays 1 and 2 interfere, and we can observe the rings in the field of view. In this case the net path difference between the rays is $\Delta = 2\mu t$. since we will not consider the path difference arises due to reflection from denser medium. Therefore this is net path difference.

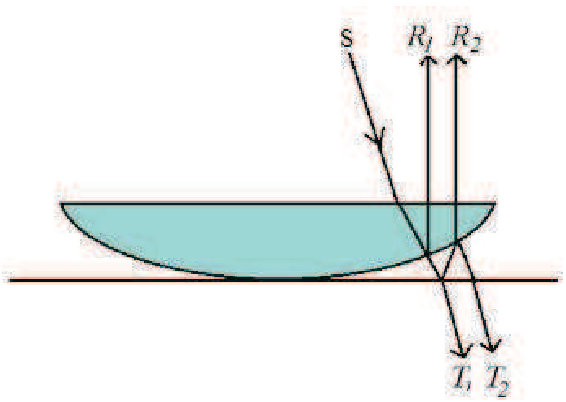


Figure 5.11

The condition for maxima (bright rings) is given by

$$2\mu t = n\lambda$$

And we know that in case of reflected light, $t = \frac{r^2}{2R}$

$$2\mu \frac{r^2}{2R} = n\lambda$$

Now if D_n is the diameter of nth bright ring then, $\frac{D_n}{2} = r$ and thus

$$Dn^2 = \frac{4n\lambda R}{\mu}$$

In case of air film, $$Dn^2 = 4n\lambda R$$

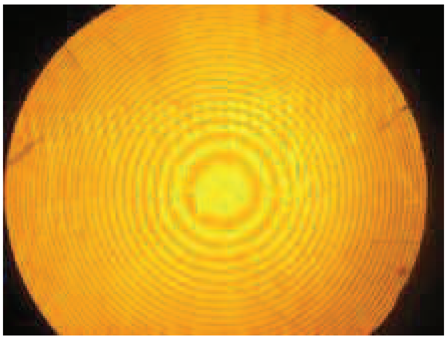


Figure 5.12

Similarly in case of minima (dark ring) the diameter nth dark ring is given by

$$D_n^2 = 2(2n-1)\,\lambda R$$

We can see that, this is an opposite case of reflected light. In case at point of contact the path difference is zero which is condition corresponding to bright fringe thus the centre point is bright. The rings system in this case is shown in figure 5.12.

5.9 SUMMARY

1. A thin film is any transparent material in a shape of thin sheet of order 1μm to10 μm. When a beam of light is incident on this sheet the interference may take place after reflection or transmission of light. In case of interference due to reflected light, the path difference

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2}$$

The condition of bright fringes (maxima)

$$2\mu t \cos r = \left(\frac{2n+1}{2}\right)\lambda \quad \text{(where n= 0,1,2,3.....)}$$

Similarly condition of dark fringes (minima) is

$$2\mu t \cos r = n\lambda \quad \text{(where n= 0,1,2,3....)}$$

2. In case of interference due to transmitted light, the path difference become $\Delta = 2\mu t \cos r$

The condition of bright fringe (maxima)

$$2\mu t \cos r = n\lambda$$

Similarly the condition of dark fringes (minima)

$$2\mu t \cos r = \left(\frac{2n+1}{2}\right)\lambda$$

3. In case of wedge shaped film the net path difference is given as

$$\Delta = 2\mu t \cos(r + \Theta) - \frac{\lambda}{2}$$

where Θ is angle of wedge and other symbols have their usual meaning. For bright fringes

$$2\mu t \cos(r + \Theta) = \left(\frac{2n-1}{2}\right)\lambda \quad \text{(where n=1, 2, 3.....)}$$

For dark fringes

$$2\mu t \cos(r + \Theta) = n\lambda \quad \text{(where n=1, 2, 3.....)}$$

If x_n is the distance of n^{th} fringe from the edge then

$$\tan\Theta = \frac{t}{x_n}$$

$$x_n = \frac{(2n-1)\lambda}{4\mu\, Cos\,(r+\theta)\tan\theta}$$

$$x_1 = \frac{\lambda}{4\mu\, Cos\,(r+\theta)\tan\theta}\quad,\quad x_2 = \frac{3\lambda}{4\mu\, Cos\,(r+\theta)\tan\theta}, \ldots\ldots\ldots\ldots$$

Fringe width $\omega = x_{n+1} - x_n$

For normal incident $r = 0^0$ and for small value of Θ ($\tan\Theta \approx \Theta$)

$$\omega = \frac{\lambda}{2\,\mu\theta}$$

4. In case of interference due to thin film the extended source of light is more beneficial. In extended source of light, a large number of rays are available for production of interference pattern and larger area of the film can be seen by our eye in the field of view.
5. On the basis of variation in two parameters *t* and *r*, the fringes are two types ray fringes of equal thickness and fringes of equal inclination.
   In case of fringes of equal thickness, the thickness of film is varying and light coming at same angle of incident then fringes are formed due to variation is thickness. The fringes are formed on the locus of points of equal thickness. Examples are thin films and Newton's rings.
   On the other hand, in case of inclination, the thickness becomes constant. Now the fringes are formed at the locus of points of constant. Such fringes are called fringes of equal inclinations. Examples are fringes formed in Michelson interferometer.
6. When a plano-convex lens of large focal length is place on a plane glass plate, an air film is formed between the lens and glass plate. When a beam of light normally incident on this film the interference takes place between the reflected rays and we observe alternate dark and bright rings and called Newton's rings.
7. In Newton's rings the condition for bright rings is given by
   $$2\mu t = \left(\frac{2n-1}{2}\right)\lambda \qquad \text{(where n=1, 2, 3….)}$$
   Similarly condition for dark rings
   $$2\mu t = n\lambda \qquad \text{(where n=1, 2, 3….)}$$
   The diameter of n^{th} bright ring is given by
   $$D^2_n = \left(\frac{2(2n-1)\lambda\, R}{\mu}\right)$$
   Similarly if D_n is diameter of n^{th} dark ring then
   $$D_n^2 = \frac{4n\,\lambda R}{\mu}$$
8. By using Newton's rings experiment, the wave length of a unknown light source can be determined as
   $$\lambda = \frac{D^2_{n+p} - D^2_n}{4pR}$$
   Where D_{n+p} is diameter of $(n+p)^{th}$ bright or dark ring and D_n is the diameter of nth bright or dark ring.
9. Newton's rings may also be observed in case of transmitted light. In this case if D_n is the diameter of n^{th} dark ring then it can be given as
   $$D_n^2 = = \left(\frac{2(2n-1)\lambda\, R}{\mu}\right)$$

Similarly if D_n is diameter of n^{th} bright ring then

$$D_n^2 = \frac{4n\,\lambda R}{\mu}$$

5.10 GLOSSARY

Thin film: A thin sheet of thickness of the order of 1-10 μm.
Wedge shaped film: A film of unequal thickness which gradually changes.
Newton's rings: Circular bright and dark fringes formed in Newton's experiment.
Narrow source: Point source
Extended source: A broader source

5.11. REFERENCE

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5.13. TERMINAL QUESTION

Short Answer Type Questions

1. Explain why different colours are exhibited by a thin film when illuminated in white light.
2. With the help of diagram explain why an extended source of light is needed to observe the interference in thin film.
3. Discuss the phase change in reflection of light from a denser medium.
4. Explain the interference in a thin film of uniform thickness.
5. Calculate the path difference between the light ray reflected from the upper and lower surface of a thin film.
6. Find out the condition of maxima and minima in reflected light in case of thin film.
7. Why a thick film does not show colours when white light is incident on it.
8. What are Newton's rings?
9. Obtain the path difference between the reflected rays in Newton's rings experiment.
10. Find out the condition for bright and dark rings in Newton's ring experiment.
11. Explain why Newton's rings are circular?
12. Explain the difference in Newton's rings formed in case of reflected and refracted light.

Long Answer Type Question

1. Discuss the formation of bright and dark fringes formed by a thin film. Explain why different colours are exhibited by thin film in white light.
2. Explain the formation of interference fringes in wedge shaped film. Obtain the condition for bright and dark fringes, and fringe width.
3. What are Newton's rings? Draw a ray diagram for Newton's rings experiment. Find out the diameter of bright and dark rings.
4. What are Newton's rings? Derive the expression for diameter of bright and dark rings.
5. Give the theory of Newton's rings and describe how the wave length of a unknown light source can be determine with the help of these rings.
6. Describe the interference fringes observed when a thin wedge shaped film is observed by reflected light. Calculate the separation between two consecutive bright and dark fringes.
7. Show that in Newton's rings experiment, the diameter of dark rings are proportioned to root of natural numbers.
8. Explain the formation of Newton's ring. How the refractive index of a given liquid can be determined with the help of Newton's rings.
9. Describe the fringes of equal thickness and fringes of equal inclination.
10. What are Haidiger's and Newton's fringes?

Numerical Type Questions

1. A beam of monochromatic light of wavelength 5890Å is incident on a thin glass plate of refractive index 1.50 with the angle of refraction in the glass plate is 60^0. Calculate the smallest thickness of the plate which will make it appears dark by reflection.
2. Light of wave length 5000Å is incident on a soap film of refractive index 1.33 at an angle 60^0. When the reflected light is observed, a dark band is seen. If the thickness of the film is 1μm, calculate the order of the fringe dark band.
3. Calculate the thickness of a wedge shaped film at a point where the 4^{th} bright fringe is observed. The experiment is performed with a light source of wavelength 5890Å.
4. A wedge shaped film of angle 6×10^{-3} degree is illuminated normally with a monochromatic light source. If the reparation between two consecutive fringes is 3.00mm, find out the wave length of light source used.
5. In a Newton's rings experiment the diameter of 5^{th} and 12^{th} dark rings are 0.42 cm and 0.726cm. The radius of curvature of plano convex lens is 2.00m. Calculate the wavelength of light source.
6. In Newton's ring experiment a light source of wavelength 5890Å is used. If the radius of plano-convex lens is 2m and water is filled between the glass plate and plano convex lens, calculate the diameter of 5^{th} dark ring.
7. A wedge shaped film is formed with air between two glass plates, which touch each other at one point and separated by a wire of diameter 0.05 mm at a distance of 15cm. If a light of wave length 6000Å is used, calculate the fringe width.

8. In Newton's rings experiment the diameter of 4^{th} bright ring is 2.52cm. If a liquid of unknown refractive index is filled in place of air between lens and plane glass plate, the diameter becomes 2.21cm. Find out the refractive index of liquid.

9. Show that in Newton's rings experiment, the difference of square of diameters of two consecutive rings remains constant.

10. Newton's rings are formed with the help of a light source of wavelength 5890Å. If the diameter of 10^{th} dark ring is 0.5cm, calculate the radius of curvature of plano convex lens.

11. A thin equiconvex lens of focal length 4m and refractive index of 1.5 is place on a plane glass plate. A light of wave length 5890Å falls normally on it. What will the diameter of 10^{th} dark ring?

Objective Type Question

1. If the thickness of the parallel film increases, the path difference

(a) increases (b) decreases

(c) remains same (d) none of these

2. When a light wave is reflected from a surface of an optically denser medium, then the phase difference involved is

(a) $\pi/4$ (b) $\pi / 2$ (c) π (d) 2π

3. When a light wave is reflected from a surface of an optically denser medium, then the path difference involved is

(a) $\lambda/4$ (b) $\lambda / 2$ (c) λ (d) 2λ

4. In case of the thin film, the condition for constructive interference in reflected light, the path difference should be equal to

(a) $2\mu t \, Cos\, r - \frac{\lambda}{2}$ (b) $\frac{\lambda}{2}$ (c) $2\mu t \, Cos\, r + \frac{\lambda}{2}$ (d) λ

5. In Newton's rings experiment the diameter of nth bright ring is given by

(a) $D_n{}^2 = \frac{2(2n-1)\lambda R}{\mu}$ (a) $D_n{}^2 = \frac{(2n-1)\lambda R}{\mu}$

(c) $D_n{}^2 = \frac{4\lambda R}{\mu}$ (d) $D_n{}^2 = \frac{2\lambda R}{\mu}$

6. The lens used in Newton's rings experiment, which is placed on a plane glass plate to trap air film is

(a) concave (b) plano convex

(c) plano concave (d) none of these

7. In Newton's rings experiment, the diameter of bright rings is proportional to

(a) odd natural numbers (b) natural numbers

(c) even natural numbers (d) square root of odd natural numbers

Answer of Numerical Type Question

1. 0.39μm, **2.** 4^{th}, **3.** 1.02 μm, **4.** 6.28×10^{-5}cm, **5.** 4.87×10^{-5}cm **7.** 0.9 mm, **8.** 1.3, **10.** 1.06 m.

11. Hint: The focal length is given as

$$\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$$

Here, $\mu = 1.5\ , R_1 = R\ and\ R_2 = \ -R \Longrightarrow R = 4\ m$

$D^2_n = 4n\lambda R$

Therefore, $D_{10} = \sqrt{4 \times 10 \times 5890 \times 10^{-10} \times 4} = 9.70\ mm$

Answer of objective Type Question

1. (a), **2.** (c), **3.** (b), **4.** (a), **5.** (a), **6.** (b), **7.** (a)

UNIT 6: MULTIPLE BEAM INTERFEROMETRY

CONTENTS

6.1 INTRODUCTION

In first unit of interference, we understood the basic principle of interference, condition required for interference and experiment like Young double slit experiment and biprism experiment which show interference. In second unit of interference, we understood different types of thin films like wedge shaped or air films which cause interference under certain conditions. Further, we understood the fringes of equal thickness and fringes of equal inclinations.

Now in this unit of interference we are going to understand different types of interferometers, especially Michelson's interferometer. In interferometer we observe the fringes occur due to equal inclination which are called Haidinger fringes. In an interferometer, we study the different techniques of fringes formation and calculate the fringe width with great accuracy. The interferometers like Michelson interferometer have a lot of significant applications in the field of optics and other branches of physics.

6.2. OBJECTIVES

After reading this unit you will be able to understand

- Interferometry
- Haidinger fringes observed in interferometers
- Michelson interferometer
- Application and significance of Michelson's interferometer

6.3. INTERFEROMETRY

Interferometry is a branch of science in which optical waves or any other electromagnetic waves are superimposed on each other and interference phenomenon occurs. Interferometry plays important role to study in the field of optics, astronomy, fiber optics, spectroscopy, cosmology, remote sensing, particle physics plasma physics, velocity measurements and bio-molecular interactions. In present unit we only discuss the optical interferometry. Interferometers are devices use for different measurement of path difference, fringe widths, refractive index and many other parameters with the help of interference phenomenon.

6.4. FRINGES OF EQUAL INCLINATION (HAIDINGER FRINGES)

Before going ahead, we should understand the fringe formation in a interferometer. As we know the interference fringes are formed due to a path difference $\Delta = 2\mu t \, Cos\, r$ between the overlapping rays. Now for a particular wavelength, the path difference may occur due to variation of thickness *t* and angle of inclination r.

$$\delta\Delta = 2\mu\Delta t \cos r + 2\mu t\ \delta\ (Cos\ r) \qquad .... (6.1)$$

In case of a film with constant thickness then variation in path difference occurs as

$$\delta\Delta = 2\mu t\ \delta\ (Cos\ r) \qquad .... (6.2)$$

Thus the path difference occurs with the variation in the angle of inclination r. If we use an extended source of light, we have a large numbers of rays comes with equal angle of inclination r, which produces a particular path difference and fringes are observed corresponding to this path difference. Such fringes are called fringes of equal inclination. In case of Michelson interferometer, the thickness of film remains constant then the fringes are formed due to equal inclination and hence called fringes of equal inclination or Haidinger fringes.

6.5 MICHELSON INTERFEROMETER

Michelson interferometer is a device used for the formation and study of interference fringes by a monochromatic light. In this apparatus, a beam of light coming from an extended source of light is divided into two parts, one is reflected part and another is refracted part after passing through a partially polished glass plate. These two beams are brought together after reflected from plane mirrors, and finally interference fringes are produced in the field of view.

6.5.1 Construction

The apparatus is shown in Figure 6.1. The main part of the apparatus is a half silvered glass plate P, on which a beam of monochromatic light is incident. The plate P inclined at an angle 45^0 with incident light as shown in figur6.1, the incident light then divided into two parts, one is reflected part and another is transmitted part. The transmitted light is then passes through another glass plate Q which is of equal thickness as of P, and parallel to plate P, this pate Q is called compensating plate. The transmitted and reflected parts of light are normally incident on two mirrors M_2 and M_1 respectively. The mirror M_1 and M_2 are perpendicular to each other as shown in figure. The mirror M_1 is fixed in a carriage and can be moved to and fro with help of a screw and micro scale. Therefore mirror M_1 is movable and the mirror M_2 is fixed. A telescope is also fixed as shown in figure. The light reflected from mirror M_1 and M_2 are superimposed and interference fringes are formed in the field of view.

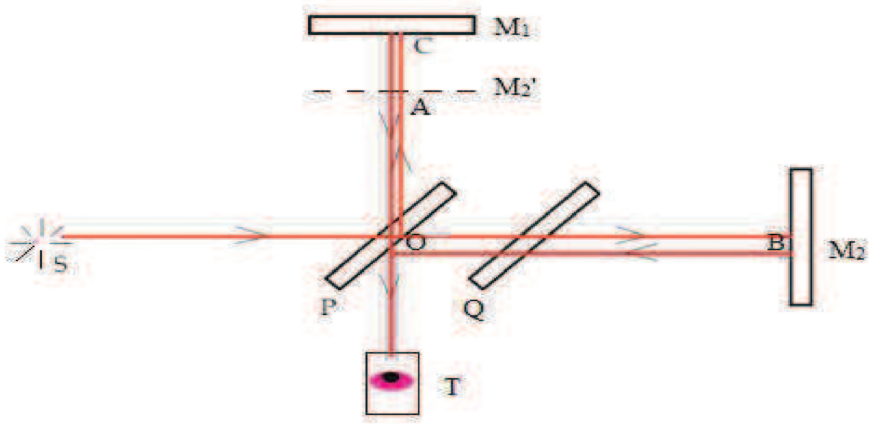

Figure 6.1

6.5.2. Working

S is a source of monochromatic light; the light coming from this source is rendered parallel by mean of a convex lens L, and after passing through Less L the light falls on plate P. Since plate P is partially polished, some part of light reflected back from P and going toward direction AC and incident on mirror M_1.

Similarly the light transmitted from plate P passing through compensating plate Q and then incident on mirror M_2. The compensating plate is used to compensate the optical path travelled by transmitted light. The beam of light reflected by P, crosses plate P two times, for transmitted light this optical path is compensated by using plate Q in which the transmitted light crosses Q two time. Thus by using compensating plate Q, the reflected and transmitted light travel equal optical path lengths.

Now the reflected light is incident on mirror M_1 and reflected back towards the telescope T. Similarly the transmitted light incident normally on mirror M_2 and reflected back towards plate P, and at P some part of this light again reflected toward the telescope. Now in the direction of telescope we have two coherent beams of light reflected from mirror M_1 and M_2, and interference takes place and we observed interference pattern/beam in the field of view.

6.5.3 Formation of Fringes

Since the fringes are form by the light reflected from mirror M_1 (movable) and M_2 (fixed) and we can consider a virtual image of M_2 called M_2' in the field of view as shown in figure 6.1. Further we can consider the interference fringes are now formed due to light reflected from the surface of air film formed between mirror M_1 and M_2'. Now it is clear that the shapes of fringes are depend upon the inclination of mirror M_1 and M_2. Since M_2 fixed therefore the shape are depends upon the inclination of M_1. Since OA = OB, therefore the path difference between two rays is simply the path traveled in air film before reaching to telescope. If t is the thickness of air film then path difference between light reflected from M_1 and M_2 is $2t$.

Condition for maxma $\Delta = 2t = n\lambda$

$$2t = n\lambda$$

If the movable mirror M_1 moved by a distance x and we observed fringes shift of N fringes then

$$2(t + x) = (n + N)\lambda$$

or $$2x = N\lambda$$

or $$\lambda = \frac{2x}{N} \quad \text{....... (6.3)}$$

It is clear that if M_1 and M_2 are exactly perpendicular to each other, then M_1 and M_2' are parallel to each other and air film between M_1 and M_2' is of equal thickness in this case we observed fringes of equal inclination or Haidinger's fringes of circular shape. If however, the two mirror M_1 and M_2 are not exactly perpendicular to each other then the shape of the air film formed between mirror M_1 and M_2' is of wedge shaped and the fringes are now of straight line parallel to the edge of wedge. This straight line fringes are because of the focus of constant thickness in a wedge shape film is a straight line.

Thus the shapes of fringes are depends on the inclination. The fringes are in general curved and convex toward the edge of wedge as shown in figure 6.2. These fringes are called localized fringes.

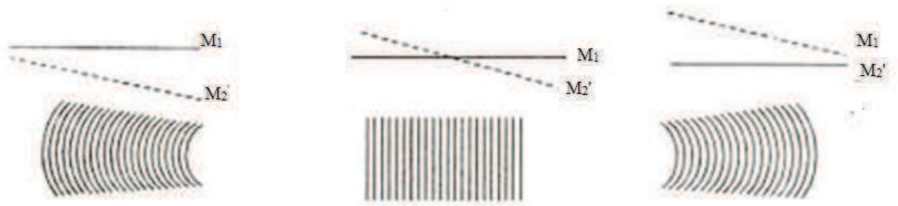

Figure 6.2

6.5.4 Determination of Difference of Wavelengths between Two Neighboring Wavelengths

Let us consider a source of light which emits two very close wavelengths. Sodium light is an example of such case. In sodium light, there are two wavelength D_1 *and* D_2 lines with wavelength λ_1 = 5890Å and λ_2 = 5896Å. By using Michelson interferometer we can determine the difference between these two wavelengths. In this case first we adjust the aperture for circular fringes. We know that each wavelength produce its own ring spectrum. Now the mirror M_1 is moved in such a way that when the position of very bright fringes are obtained. In this position the bright fringes due to λ_1 coincident with the bright fringes due to λ_2 and we observe distinct fringes of order n.

Now the mirror M_1 is further moved to a very small displacement, and the fringes are disappeared. This case occurs when the maxima due to λ_1 coincident on minima due to λ_2. This is the position of minimum intensity or uniform illumination with no clear fringes. In this case we observed indistinct fringes of order (n+1). If we moved a distance x between such two points of most bright and most indistinct fringes then

$$2x = n\lambda_1 = (n+1)\lambda_2$$

or $$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

or $$2x = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

or $$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2x} \qquad \text{....... (6.4)}$$

If λ_1 and λ_2 are very close to each other then

$$\lambda_1.\lambda_2 = \lambda^2$$

Where λ is the mean value of λ_1 and λ_2

$$\lambda = \frac{\lambda_1 + \lambda_2}{2}$$

Then $$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2x} \qquad \text{....... (6.5)}$$

6.5.5 Determination of Refractive Index of a Material

In Michelson interferometer, the two interfering beam of light travel in different directions, one is toward mirror M_1 and second one is toward mirror M_2. It is very easy to introduce a thin transparent sheet of a material of refractive index is and thickness t, in the path of one of the interfering beams of light. After introducing a sheet, the optical path of that beam increases by μt. Now the net increase in the path is $(\mu t - t)$. Since the beam crosses the sheet twice, the net path difference becomes $2(\mu t - t)$.

If n is the number of fringes be which the fringe system is displaced, then

$$2(\mu t - t) = n\lambda$$

or $$2(\mu - 1)t = n\lambda \qquad \text{....... (6.6)}$$

In experiment we first locate the central dark fringe by using while light. The cross wire of telescope is adjusted in such a way that the cross wire of telescope is adjusted on central dark fringes. Now the light is replaced by a monochromatic light of wavelength λ. Now a thin sheet is introduced into the path of one beam. The position of movable mirror M_1 is adjusted in such a way that the dark fringe is again coincide with the cross wire of telescope. We note the distance d through which the mirror is moved and count number of fringes displaced. By using the relation given below we can determine the thickness of sheet.

$$t = n\lambda / 2(\mu - 1) \qquad \text{....... (6.7)}$$

Similarly if we know the thickness, we can determine the refractive index of material.

$$2(\mu - 1)\, t = n\lambda$$

$$\mu = (n\lambda / 2t) + 1 \qquad \text{....... (6.8)}$$

6.5.6 Michelson Morley Experiment and Its Result

In classical mechanics it was assumed that the preferred medium for light propagation is ether which filled in all space uniformly. The ether is perfectly transparent medium of light and material bodies may pass in this medium without any resistance. Ether remains fixed in space and consider as absolute frame of reference. In the 19^{th} century this ether drag hypothesis of light was widely discuss.

Michelson interferometer was originally designed to verify the existence of hypothetical medium ether. The experiment performed to verify this hypothesis is called Michelson Morley experiment. In this experiment, it was assumed that the Michelson interferometer is moving along the earth direction of motion. Due to motion of apparatus with transmitted light are not same. Mathematically the path difference between two ray (transmitted and reflected) is lv^2/c^2 where l is distance between plate P and mirror M_1 and v is velocity of ether corresponding to this path difference there should be a fringe shift of n = 0.37. Thus if the apparatus is at rest and starts motion, there should be a fringe shift of n = 0.37. But it is not possible to make earth at rest. In this experiment we consider if the whole apparatus was turned by 90^o, the fringe shift should be observed.

The experiment was performed by many scientists, many times at different location on earth but fringe shift was not observed. This is called negative result of Michelson Morley experiment. The result shows the non existence of hypothetical medium of ether. After this experiment, a foundation of modern though way lay down which led to Einstein theory of relativity.

Self Assessment Questions

1. What is an interferometer?
2. What is the role of compensating plate in Michelson interferometer?
3. How the air film is formed in Michelson's interferometer?
4. How the path difference is calculated in Michelson's interferometer?
5. Why fringes are circular in Michelson's interferometer?
6. What is the meaning of localized fringes?
7. What happens when white light is used in Michelson's interferometer?
8. Determine the thickness of a thin transparent film with the help of Michelson's interferometer.
9. Determine the refractive index of a material with the help of Michelson's interferometer.
10. If the mirrors M1 and M2 of Michelson's interferometer are exactly perpendicular to each other, how will be the shape of fringes?
11. How you will find the wavelength of a monochromatic light with Michelson's interferometer.
12. Give the application of Michelson's interferometer.

6.6 SOLVED EXAMPLES

6.1. In Michelson interferometer, when movable mirror M_1 is shifted by a distance 0.030mm, a fringe shift of 100 fringes is observed. Calculate the wavelength of light used.

Solution: In Michelson interferometer if the mirror is displaced by a distance x, the corresponding fringe shift N is

$$2x = N\lambda \quad \text{or} \quad \lambda = 2x/N = 2(0.030)/100 = 6000\text{Å}$$

6.2. The difference between two wavelengths of sodium light lines $D_1\ and\ D_2$ is determined with the help of Michelson intereferometer. If the distance travelled by movable mirror for two successive position of most distinct and most indistinct position is 0.2945 mm calculate the difference between two wavelengths D_1 *and* D_2, the mean wavelength of two lines is 5893Å

Solution: If the displacement between two position of mirror for two successive position of most distinct and most indistinct position is x then

$$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2x} = (5893 \times 5893)/(2 \times 0.2945 \times 10^7) = 6Å$$

6.3. Reflective index of a glass plate is to be determined by the help of Michelson interferometer. If is observed that when the glass plate is introduced, a fringe shift of 140 is observed. If the length of glass plate is 20cm and the wavelength of light is 5460Å, calculate the refractive index of material.

Solution: when a glass plate is introduce in one of the interfering ray of Michelson's interferometer then a fringe shift is observed as

$2(\mu - 1)\, t = n\lambda$ or $\mu = (n\lambda/2t) + 1 = [(140 \times 5460 \times 10^{-10}) \div (2 \times 20 \times 10^{-8})] + 1 = 1.0029$

6.4. In Michelson interferometer 790 fringes cross the field of view when the movable mirror is displaced through a distance 0.233mm. Calculate the wavelength of light used.

Solution: In Michelson interferometer if movable mirror is displaced through a distance x, the corresponding fringe shift n is given as

$$2x = n\lambda \quad \text{or} \quad \lambda = 2x/n = 2\times 0.233/790 \text{ mm} = 5896Å$$

6.7 SUMMARY

1. Interferometer is a device used for measurement of path difference, fringe width, refractive index, wavelength of a monochromatic light source and many other parameters with the help of interference phenomenon.
2. In Michelson's interferometer, an air film is formed with the help of two perpendicular mirrors. The light reflected from two mirrors M_1 and M_2 is equivalent to light reflected from the upper and lower surface of air film formed between mirror M_1 and M_2'.
3. The condition for bright fringes is given as $2x = N\lambda$
   Where, x = displacement of mirror M_1, N = number of fringe shifts on displacement of x, λ = wavelength of light used.
4. In Michelson interferometer if M_1 and M_2 mirror are exactly perpendicular to each other, the shape of fringes are circular which are called fringes of equal inclination or Haidinger fringes. If however, two mirrors are not perpendicular to each other, the shape of film formed between M_1 and M_2' is of wedge shape and the fringes are straight line or localised.
5. The difference between two neighboring wavelength of a source is given as
   $$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2x}$$
6. The refractive μ index of a medium can be determine by
   $2(\mu - 1)\, t = n\lambda$ or $\mu = (n\lambda/2t) + 1$
7. The thickness t can be determine by, $t = n\lambda/2(\mu - 1)$

6.8 GLOSSARY

Interferometer: A device used for measurement of path difference, fringe width, wavelength of light, refractive index etc. with the help of interference phenomenon.

Inclination: Degree of sloping, slope

Haidinger fringes: The fringes of equal of inclination.

Compensating plate: A plate used in Michelson interferometer for compensating the path difference in transmitted light raised due to glass plate.

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6.11 TERMINAL QUESTIONS

Short Answer Type Questions

1. Describe the construction of Michelson interferometer.
2. Describe the working of Michelson interferometer.
3. How Michelson's interferometer may be used to obtain circular and streight line fringes.
4. Explain why circular fringes shift in the field of view when we move the mirror M_1.
5. Outline the theory of Michelson's interferometer.
6. With the help of Michelson interferometer how the D_1 and D_2 lines of sodium light can be distinguished. Find out the difference between D_1 and D_2 lines of sodium light.
7. How the refractive index of a medium can be determined with the help of Michelson interferometer.
8. Explain the method of determine the thickness of sheet/fill with the help of Michelson interferometer.
9. Explain the role of compensation plate in Michelson's interferometer.
10. What are localised fringes in Michelson's interferometer?
11.

Long Answer Type Questions

1. 1 With the help of neat diagrams, describe the construction and working of Michelson's interferometer.
2. Explain the working of Michelson's interferometer. How the interferometer produces straight line and circular fringes.
3. Give the applications of Michelson's interferometer in detail.
4. Explain how circular fringes are produced in Michelson's interferometer. Show that the radii of circular fringes obtained by the Michelson's interferometer are proportional to the square root of natural number.

Numerical Type Questions

1. 1. Calculate the displacement between two successive positions of movable mirror giving the best fringes in case of sodium light. [Answer: 0.029cm]
2. 2. In Michelson's interferometer when movable mirror is displaced through a distance 0.589mm, a fringe shift of 200 is observed across the cross wire in the field of view. Calculate the wavelength of light used. [Answer: 5890A]
3. Determine the difference between the wavelengths of two D_1 and D_2 lines in sodium light. The wavelengths of D_1 and D_2 lines are 5896 Å and 5890 Å respectively. The scale reading of two successive distinct and indistinct points are 0.6939mm and 0.9884mm. [Answer: 6Å]
4. In Michelson's Interferometer when movable mirror is displaced through a distance 0.844mm a fringe shift of 300 is observed. Calculated the wave length of light used. [Answer: 562Å]
5. 3. Determine the difference between the wavelengths of two D_1 and D_2 lines in sodium light. The wave length of D_1 and D_2 are 5896 Å and 5890 Å respectively. The scale readings of two successive distinct and indistinct points are 0.6939mm and 0.9884mm. [Answer: 6 Å]
5. In Michelson's Interferometer when movable mirror is displaced through a distance 0.844mm a ping shift of 300 is observed. Calculated the wave length of light used.

   [Answer: 562 Å]

Objective Type Questions

1. In Michelson interferometer, when mirror M_1 and M_2 are perpendicular to each other, then the shape of the fringes are
   (a) Straight line (b) Circular
   (c) elliptical (d) inclined
2. The use of compensating plate in the Michelson Interferometer is
   (a) To make path difference equal between light beams reflected from mirror M_1 and M_2
   (b) To make frequency equal between light beams reflected from mirror M_1 and M_2
   (c) To make path difference $\frac{\lambda}{2}$ between light beams reflected from mirror M_1 and M_2

(d) To make path difference λ between light beams reflected from mirror M_1 and M_2

[Answers 1(b), 2(a)]

UNIT-7: DIFFRACTION OF LIGHT WAVES AND FRESNEL DIFFRACTION

CONTANTS

7.1 INTRODUCTION

In the preceding units we have read, that the interference phenomenon arises when two or more coherent light beams, obtained either by division of wavefront or by division of amplitude, meet each other. In this unit we shall discuss the interference effect of secondary wavelets originating from the same wavefront or from single aperture. This is called diffraction. The wave nature of light was further confirmed by the phenomenon of diffraction.

Diffraction refers to various phenomena which occur when a wave encounters an obstacle or a slit (or aperture). Since at the atomic level, physical objects have wave-like properties, they can also exhibit diffraction effects. The diffraction of light was first observed and characterized by an italian mathematician Francesco Maria Grimaldi. The word diffraction originated from Latin word ‘diffractus’ which means ‘to break into pieces’. Thus he referred this phenomenon as breaking up of light into different directions. Isaac Newton attributed them to inflexion of light rays. James Gregory used a bird feather and observed the diffraction patterns. This was effectively the first diffraction grating to be discovered. Augustin-Jean Fresnel did more studies and calculations of diffraction and thereby gave great support to the wave theory of light that had been advanced by Christiaan Huygens.

The effects of diffraction are often seen in everyday life. For example, the closely spaced tracks on a CD or DVD act as a diffraction grating for incident light and form a rainbow like pattern when seen at it. The hologram on a credit card is another example. Almost the same colourful pattern is formed due to the diffraction of light. A bright ring around a bright light source like the sun or the moon is because of the diffraction in the atmosphere by small particles.

7.2 OBJECTIVES

Upon completion of this unit you will be able to

- State the diffraction of light and the necessary conditions for producing this effect
- Differentiate the phenomena interference and diffraction
- Describe the Fresnel and Fraunhofer classes of diffraction
- Define the construction of half period zones and compute their radii and area
- Find the resultant amplitude at a point on the screen due to a number of zones
- Prove that the light propagate along a rectilinear path
- Describe a zone plate, its construction, its action and theory.
- List the similarities and dissimilarities between a zone plate and a lens
- To understand ‘what kind of diffraction effect is produced by a sharp straight edge at various points on the screen’
- Find the expressions for the positions of maxima and minima, and the intensity distribution due to diffraction effect produced by a sharp edge

7.3 DIFFRACTION OF LIGHT

As per the rules of geometric optics, the light should caste a well defined and distinct shadow of an object placed in its path. If the direction of incidence of light is perpendicular to the length of obstacle then due to its rectilinear propagation, the size of the image should be equal to the size of the object (fig. 7.1). No light should reach into the regions of shadow. The same thing happens with aperture. Light enters from the open region of aperture and reaches to the screen (fig.7.2). When the direction of incidence is not normal to length of obstacle (or aperture), the size of image (or shadow) will be different from that of obstacle or aperture (fig.7.3 and 7.4).

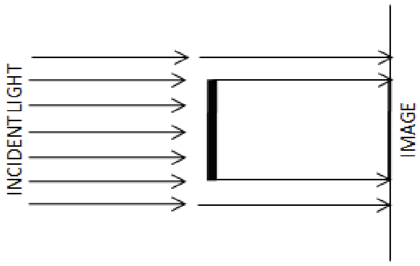

Figure 7.1

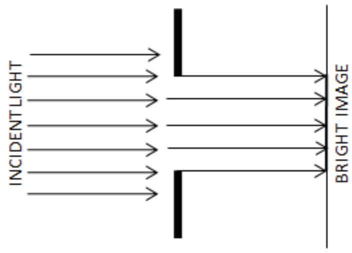

Figure 7.2

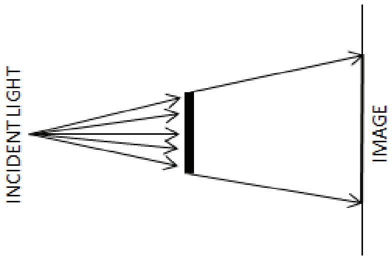

Figure 7.3

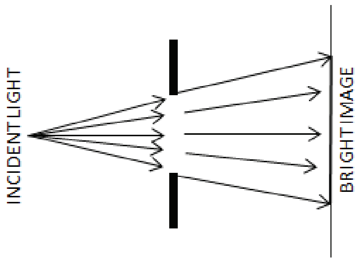

Figure 7.4

A very close and careful observation of light distribution reveals that there are dark and bright fringes near the edges. As the size of the aperture is decreased the fringes become more and more distinct. When the size of aperture becomes comparable to the wavelength of incident light the fringes become broad and practically cover the entire shadow region, so instead of a sharp shadow we obtain bright and dark fringes on the screen.

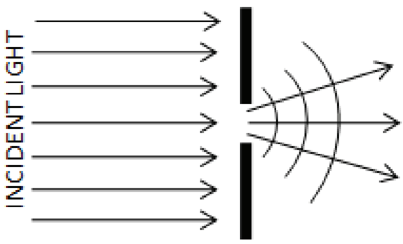

Figure 7.5

In simple language we can say that ‘when the size of the opaque obstacle (or aperture) is small enough and is comparable to the wavelength of incident light, the light bends round the corners’. If the opening is much larger than the light's wavelength, the bending will be almost unnoticeable. The phenomenon of bending of light round the corner or edge and spreading into the geometrical shadow region of the obstacle (or aperture), placed in its path, is known as diffraction. The bending of light for a small slit is shown in figure 7.5. The formation of alternate bright and dark fringes, by the redistribution of light intensity, is called the diffraction pattern. The amount of bending depends on the relative size of the wavelength of light to the size of the opening.

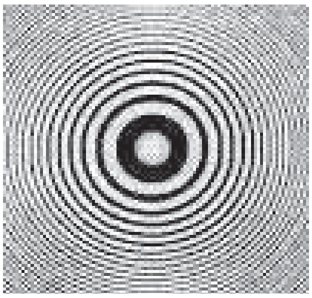

Figure 7.6

Dominique Arago placed a small circular disc in between a point light source and screen and obtained almost a regular pattern of alternate dark and bright rings. There was a bright circular spot at the centre of this pattern. The formation of this kind of diffraction pattern could not be explained on the basis of rectilinear propagation of light. Thus wave theory of light was used to explain the bending of light into the regions of geometrical shadow. One such pattern is depicted in figure 7.6.

Self Assessment Question (SAQ) 1: What do you understand by the term diffraction? What is the condition of obtaining observable diffraction pattern?

7.4 DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION

(i) The interference occurs between two separate wavefronts originating from two coherent sources while in the phenomenon of diffraction the interference occurs between the secondary wavelets originating from different points of the exposed part of same wavefront.

(ii) In the interference pattern all the maxima are of the same intensity but in diffraction pattern the intensity of central maximum is maximum and goes on decreasing as we move away.

(iii) The interference fringes are usually equally spaced while the diffraction fringes are never equally spaced.

(iv) In interference the minima are perfectly dark but it is not so in diffraction pattern.

7.5 FRESNEL AND FRAUNHOFER CLASSES OF DIFFRACTION

The diffraction phenomenon is usually divided into two classes; the Fresnel diffraction and Fraunhofer diffraction. Following are the main differences between these two types of diffractions.

(i) In Fresnel diffraction either the source of light or the screen or both are in general at finite distance from the diffracting element (obstacle or aperture) whereas in Fraunhofer diffraction both the source of light and the screen are at infinite distance from diffracting element.
(ii) In Fresnel diffraction no lenses are used for rendering the rays parallel or convergent therefore the incident wavefront is divergent either spherical or cylindrical. In Fraunhofer class of diffraction generally two convergent lenses are used; one to make the incoming light parallel and other to focus the parallel diffracted rays on the screen. The incident wavefront is, therefore, plane.
(iii) In Fresnel diffraction the phase of secondary wavelets is not the same at all points in the plane of aperture while converse is true for Fraunhofer diffraction.
(iv) Depending on the number of Fresnel's zones formed, the centre of the diffraction pattern may be either dark or bright in Fresnel diffraction but in Fraunhofer diffraction it is always bright for all paths parallel to the axis of lens.
(v) In Fresnel class of diffraction the lateral distances are important while in Fraunhofer diffraction the angular inclination plays important role in the formation of diffraction pattern.
(vi) In Fresnel diffraction the diffraction pattern formed is a projection of diffracting element modified by the diffracting effects and the geometry of the source and in Fraunhofer diffraction the diffraction pattern is the image of the source modified by the diffraction at diffracting element.

SAQ 2: How will you differentiate the interference and diffraction phenomenon?
SAQ 3: Write any four differences between Fresnel and Fraunhofer class of diffraction.

7.6 FRESNEL'S HALF PERIOD ZONES

According to Huygens principle each point on a wavefront acts as a source of secondary disturbance. When a wavefront is made to incident on a slit, most of it is obstructed by the slit. The small portion of the wavefront passed through the slit is, thus, equivalent to a string of coherent point sources. The intensity at any point on the screen may be obtained by suitably summing the intensities of wavelets originating from those point sources at the slit and superposing at that point of screen. Thus diffraction pattern is formed at screen due to the interference of secondary wavelets.

Since the coherent sources are located at different distances from any point on the screen, the waves reach that point with differing phases. Their superposition produces interference pattern with maxima and minima formation. Therefore, the diffraction of light is due to the superposition of waves from coherent sources of the same wavefront after the wavefront is obstructed by obstacle or aperture.

7.6.1. Construction of Zones

For the qualitative understanding of the diffraction pattern, Fresnel introduced the idea of half period zones. The wave-front originated from the source and striking the obstacle or aperture is divided into a number of the circular and the concentric zones. Zone is the small area on the plane wave-front with reference to the point of the observation such that all the waves from the area reach the point without any path difference. The paths of light rays from the successive zones differ by *λ/2*. Since path difference of *λ/2* corresponds to half time period, these zones are known as half period zones.

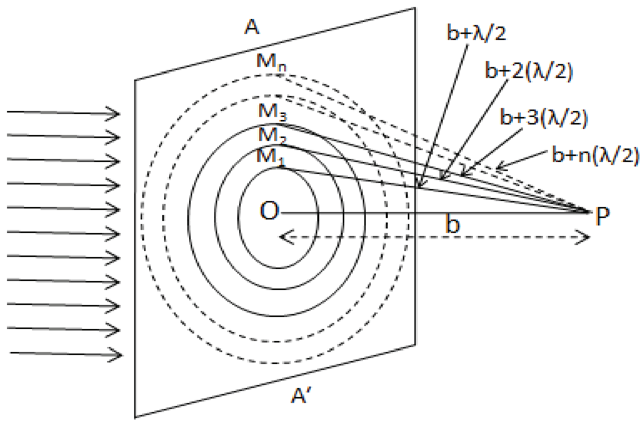

Figure 7.7

In order to understand the construction of half period zones taking a plane wavefront AA' and droping a perpendicular PO on the wavefront from an external point P. If the distance PO is *b* then taking *P* as a centre draw spheres of radii *b+λ/2*, *b+2(λ/2), b+3(λ/2) etc.* The spheres will cut the wavefront AA' in circles of radii OM_1, OM_2, OM_3 etc as shown in figure 7.7. The annular regions between two consecutive circles are called *half period zones,* e.g., the annular region between $(n\text{-}1)^{th}$ circle and n^{th} circle is called the n^{th} half period zone.

7.6.2. Radii and Area of Zones

From simple geometry the radius of n^{th} such circle, OM_n, can be written as

$$OM_n = r_n = \left[\left(b + n\frac{\lambda}{2}\right)^2 - (b^2)\right]^{1/2}$$

$$= \sqrt{n\lambda b}\left[1 + \frac{n\lambda}{4b}\right]^{1/2} = \sqrt{n\lambda b} \qquad \text{....... (7.1)}$$

Here we have assumed *b>> λ,* which is true in most of the experiments using visible light. We have also assumed here that *n* is not a very large number. From expression given by equation (7.1), it is clear that the radii of half period zones are proportional to the square roots of natural numbers. Therefore, the radii of first, second, third etc. half period zones are $\sqrt{\lambda b}$, $\sqrt{2\lambda b}$, $\sqrt{3\lambda b}$ etc

With the help of equation (7.1), the area of n^{th} half period zone is given by

$$A_n = \pi r_n^2 - \pi r_{n-1}^2 = \pi[n\lambda b - (n-1)\lambda b] = \pi\lambda b \qquad .... (7.2)$$

Thus for $b >> \lambda$ and n not very large, the areas of half period zones are independent of n and are approximately equal for fixed value of λ and b. The area of the zone may be varied by varying the wavelength of light used and the distance of the point from the wavefront.

Example 7.1. A screen is placed at a distance of 100 cm from a circular hole illuminated by a parallel beam of light of wavelength 6400 Å. Compute the radius of fourth half period zone.

Solution: If b is the distance of the point of consideration from the pole on the wavefront then the radii of the spheres whose sections cut by by the wavefront from the half period zones are $b + \frac{\lambda}{2}$, $b + \frac{2\lambda}{2}$, $b + \frac{3\lambda}{2}$ etc. Hence the radius of fourth half period zone is given by

$$r_4 = \sqrt{\left(b + \frac{4\lambda}{2}\right)^2 - b^2} = \sqrt{(4\lambda^2 + 4b\lambda)} \cong \sqrt{4b\lambda}. \text{ Because } 4b\lambda \gg \lambda^2$$

It is given that, $b = 100\ cm$ and $\lambda = 6400 \text{ Å} = 6400 \times 10^{-8}\ cm$

∴ $r_4 = \sqrt{4 \times 100 \times 6400 \times 10^{-8}} = 0.16\ cm$

Example 7.2: A plane wavefront of light of wavelength 1000 Å is allowed to pass through an aperture and a diffraction pattern is obtained on the screen placed at a distance of 1m from aperture find the radius and area of 1000th half period zone.

Solution: Given that $\lambda = 1000 \text{x} 10^{-10}$ m = 10^{-7} m, b = 1m and n =1000

We know that the radius of nth zone is given by, $r_n = \sqrt{nb\lambda}$

∴ $r_{1000} = \sqrt{1000 \times 1 \times 10^{-7}} = 10^{-2}\ m = 1.0\ cm$

The area of zone = $\pi b\lambda$ = 3.14x1x10^{-7} = 3.14x10^{-7} m^2

Example 7.3: A light of wavelength 5x10^{-7} m is made to incident on a hole. Calculate the number of half period zones lying within the hole with respect to a point at a distance of 1.0 m from the hole if the radius of hole is (i) 10^{-3} m and (ii) 10^{-2} m.

Solution: It is given that λ = 5x10^{-7} m, b = 1 m. If A_n is the area of hole of radius r_n containing n-half period zones each of area πbλ then, we have, $A_n = \pi r_n^2$ = n. πbλ

(i) For $r_n = 10^{-3}$ m,

Substituting in the above equation, we get,

$$\pi \text{ x } (10^{-3})^2 = \text{n x } \pi \text{ x 1 x 5 x } 10^{-7}$$

∴ $$n = \frac{10^{-6}}{5 \times 10^{-7}} = 2$$

(ii) For $r_n = 10^{-2}$ m

$$\pi \text{ x } (10^{-2})^2 = \text{n x } \pi \text{ x 1 x 5 x } 10^{-7}$$

$$\therefore \qquad n = \frac{10^{-4}}{5 \times 10^{-7}} = 200$$

SAQ 4: The radius and area of nth zone are 1.0 cm and 3.14x10⁻⁷ m². Find the value of n.

SAQ 5: Light of 5000 Å is passed through a hole and two half period zones are formed with respect to a point at a distance of 1.0 m from the hole. Calculate the diameter of the hole.

7.6.3. Resultant Amplitude at Point P

According to Fresnel the resultant amplitude at any point due to whole of the wavefront will be the combined effect of all the zones, while the amplitude produced by a particular zone is proportional to the area of the zone and inversely proportional to the distance of the zone from the point of consideration, *P*. This amplitude also varies with obliquity factor $\frac{1}{2}(1 + cos\theta)$. Where θ is the angle between the normal *PO* to the wavefront and the line *QP*. Thus if u_n represents the amplitude produced by the secondary wavelets emanating from the n^{th} zone then we can write

$$u_n = (Constant) \times \frac{A_n}{Q_n P} \times \frac{(1+cos\theta_n)}{2} \qquad \text{....... (7.3)}$$

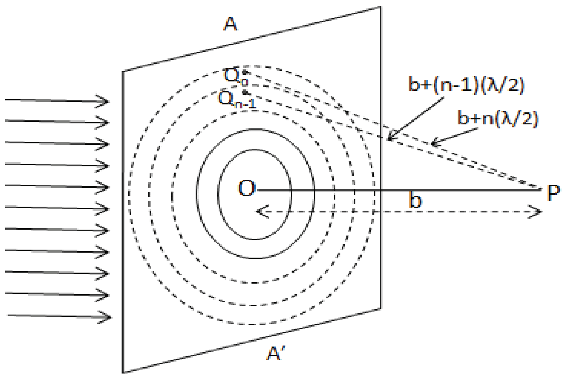

Figure 7.8

Where θ_n is the value of θ for n^{th} zone. If we take infinitesimal areas around point Q_n in the n^{th} half period zone and around a corresponding similar point Q_{n-1} in $(n-1)^{th}$ half period zone as shown in the figure 7.8 such that

$$Q_n P - Q_{n-1} P = \lambda/2 \qquad \text{....... (7.4)}$$

This path difference of $\lambda/2$ corresponds to a phase difference of π. Although the areas of the zones are almost the same but the distance of the zone from point *P* and the value of θ increases as we move from lower to higher *n*. The amplitudes u_1, u_2, u_3 etc. of 1st, 2nd, 3rd etc. zones at point *P* will be, therefore, in gradually decreasing order as shown in figure 7.9. The opposite directions of alternate amplitudes correspond to the phase change of π between consecutive zones.

Thus the resultant amplitude at *P* can be written as

$$u_p = u_1 - u_2 + u_3 - u_4 + \ldots\ldots\ldots + (-1)^{n+1} u_n \qquad \ldots\ldots\ (7.5)$$

The positive and negative signs on the right hand side between alternate terms of this equation may be ascribed to the fact that the disturbances produced by two consecutive zones at *P* will be out of phase by π radians.

As the disturbances at *P* due to various zones are of gradually decreasing magnitudes, the amplitude due to any zone may be taken approximately equal to the average of the amplitudes due to the preceding zone and the succeeding zone. That is, we can take

$$u_2 = \frac{u_1+u_3}{2},\ u_4 = \frac{u_3+u_5}{2} \text{ etc.} \qquad \ldots\ldots\ (7.6)$$

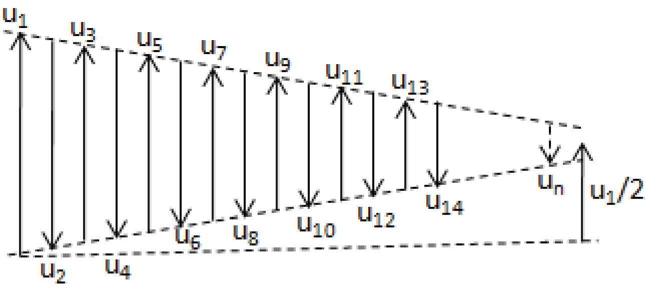

Figure 7.9

In equation (7.5), the last term on right hand side will be positive if *n* is odd and negative if it is even. We can rewrite equation (7.5) as

$$u_p = \frac{u_1}{2} + \left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right) + \left(\frac{u_3}{2} - u_4 + \frac{u_5}{2}\right) + \ldots \qquad \ldots\ldots\ (7.7)$$

Thus if *n* is odd we have, $u_p = \frac{u_1}{2} + \left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right) + \ldots\ \ldots + \left(\frac{u_{n-2}}{2} - u_{n-1} + \frac{u_n}{2}\right) + \frac{u_n}{2}$

Using equation (7.6), we get, $u_p = \frac{u_1}{2} + \frac{u_n}{2}$ .... (7.8)

And if *n* is even then,

$$u_p = \frac{u_1}{2} + \left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right) + \ldots\ \ldots + \left(\frac{u_{n-3}}{2} - u_{n-2} + \frac{u_{n-1}}{2}\right) + \frac{u_{n-1}}{2} - u_n$$

Using equation (7.6), we have, $u_p = \frac{u_1}{2} + \frac{u_{n-1}}{2} - u_n$ .... (7.9)

If the number of half period zones formed is large enough then due to gradually decreasing amplitudes of zones, the values of u_n and u_{n-1} may be neglected as compared to u_1, and therefore we can write

$$u_p \cong \frac{u_1}{2} \qquad \ldots\ldots\ (7.10)$$

And the intensity at point *P*, therefore, may be given by

$$I_p \cong u_p^2 = \frac{u_1^2}{4} \qquad \ldots\ldots\ (7.11)$$

Thus the resultant amplitude produced by whole of the wavefront is equal to one half of that produced by the first zone and the intensity due to the entire wavefront is the one fourth of that by the first zone.

Example 7.4: A plane wavefront of light of wavelength $5x10^{-5}$ cm falls on a circular hole and is received at a point 200 cm away from that hole. Calculate the radius of the hole so that the amplitude of light on the screen is two times the amplitude in the absence of hole.

Solution: It is given that $\lambda = 5x10^{-5}$cm = $5x10^{-7}$ m and b = 200 cm = 2.0 m

We know that the amplitude due to the whole wavefront is only half to that due to first half period zone, therefore

Radius of hole = Radius of first half period zone = $\sqrt{b\lambda} = \sqrt{(2.0 \times 5 \times 10^{-7})} = 10^{-3}m = 1.0\ mm$

SAQ 6: The radius of an opening is $4.47x10^{-2}$ cm. The light of wavelength λ is passed through that opening and collected at a distance of 40 cm from opening. Calculate the wavelength of light so that the intensity of light on the screen is four times the intensity in the absence of the opening.

7.7. RECTILINEAR PROPAGATION OF LIGHT

With the help of the theory discussed so far we can explain the rectilinear propagation of light. Suppose a plane wavefront of monochromatic light is made to incident on a screen with square aperture *ABCD* and whole of the wavefront except *ABCD* portion is blocked by the screen as shown in the figure 7.10. Let *P* be a point at which the intensity of the light is required and its pole *O* with respect to the aperture *ABCD* is well inside from the edges. Taking *O* as centre if we draw the half period zones in the incident wavefront then the number of the wavefronts will be quite large before they intersect the edges *AB*, *BC*, *CD*, and *DA*. Thus practically all the effective zones are exposed and the resultant amplitude at *P* due to aperture *ABCD* is given by equation (7.10). This amplitude is equal to the one half that due to the first zone and since the areas of these zones are extremely small, we can consider the light to be travelling along a straight line along *OP*. This condition is the same as if the screen with square aperture *ABCD* was removed.

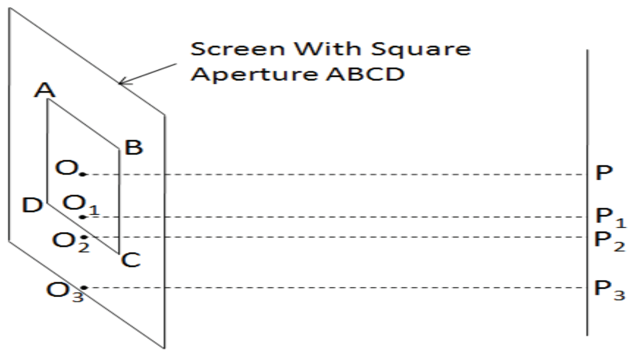

Figure 7.10

The poles O_1 and O_2 of the points like P_1 and P_2 on the screen lie very close to edges of the aperture *ABCD*. If we draw the half period zones around these poles then some of the zones are obstructed and some are exposed. Thus there will be neither uniform illumination nor complete darkness at points P_1 and P_2. For the points near the edges the light, therefore, enters into the geometrical shadow region. The point P_3 is well inside the geometrical shadow region and its pole is O_3. Since the amplitude at a point due to a zone decreases on increasing its order, almost all the effective zones around O_3 are cut off. The amplitude reaching at P_3 is nearly zero and there is a complete darkness. This is possible only when light travels along a straight line.

From the above mentioned facts this may be concluded that there is almost uniform illumination at the points whose poles lie well inside the edges of the aperture and complete darkness at the points whose poles lie well outside the edges. This strongly supports the rectilinear propagation of light. There is a slight deviation from the rectilinear path for the points whose poles lie very close to the edges. However due to very small value of the wavelength of light this region is very small as compared to whole of the aperture. Thus as a whole the propagation of the light may be considered along a rectilinear path.

7.8. ZONE PLATE

A zone plate is a device used to focus light; however zone plates use diffraction instead of refraction or reflection as in case of lenses and curved mirrors. It is a specially designed diffraction screen consisting of a large number of half period zones. In the honor of Augustin-Jean Fresnel they are sometimes called Fresnel zone plates. It is constructed in such a way that every alternate zone blocks the light incident on it. In other words we can say that it consists of alternate opaque and transparent set of radially symmetric rings (zones).

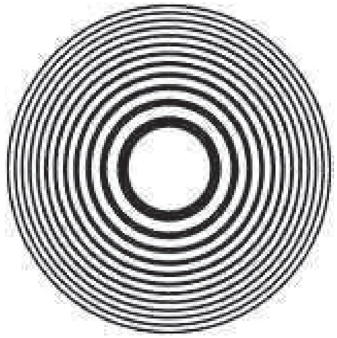

Figure 7.11

Figure 7.12

The zones can be spaced so that the diffracted light constructively interferes at the desired focus. The light may be cut off either by even numbered zones or by odd numbered zones. When the light is obstructed by even numbered zones the plate is known as positive zone plate and when obstructed by odd numbered zones it is called negative zone plate. These two kinds of zone plates are shown in figures 7.11 and 7.12.

7.8.1. Construction and Theory of Zone Plate

From equation (7.1) of section 7.6.2, it is evident that the radii of half period zones are proportional to square roots of natural numbers. Thus to construct a zone plate, we draw the concentric circles of the radii proportional to square roots of natural numbers on a white paper. The alternate regions between the circles are painted black. If the odd numbered zones are painted black then drawings appears like figure 7.12 and if even numbered zones are covered with black ink then the drawing looks like figure 7.11. Suppose the drawing resembles with figure 7.11. If we take a reduced photograph of it then the developed negative resembles with figure 7.12. This negative is then used as a zone plate.

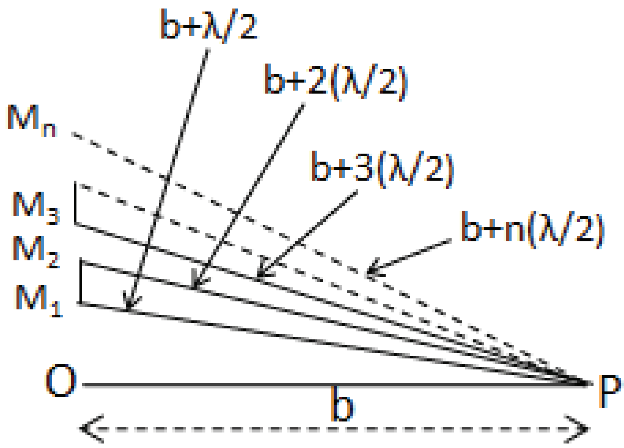

Figure 7.13

If a beam of light is made to incident on such a zone plate normally and a screen is placed on the other side of this plate to get an image then the maximum brightness is obtained at a particular point of the screen. Suppose this point is *P* at a distance of *b* units from the zone plate as shown in figure 7.13. Only upper half portion of the zone plate is shown in this figure. If λ is the wavelength of light used then radius of the first zone ($OM_1=r_1$), second zone ($OM_2=r_2$) etc are given by $r_1 = \sqrt{b\lambda}$ and $r_2 = \sqrt{2b\lambda}$ etc.

The general expression for radius may be written as

$$r_n = \sqrt{nb\lambda} \text{ or } b = \frac{r_n^2}{n\lambda} \qquad \text{....... (7.12)}$$

Since the wavelength of light has a small value, the sizes of the zones are usually very small as compared to the distance of the light source from the zone plate. Hence OM_1, OM_2, OM_3 etc are extremely small as compared to distance *a* (source *S* to zone plate *AB* separation). But to make the points M_1, M_2, M_3 etc distinct and to show the complete figure the distances are not taken in this ratio in figure 7.14. Because of this reason the incident wavefront may be taken as a plane wavefront.

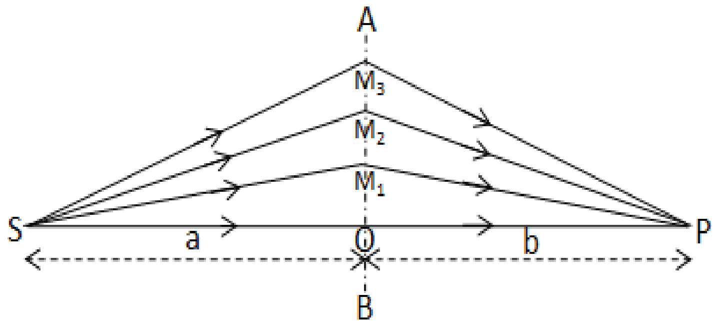

Figure 7.14

Now suppose even numbered zones are opaque to incident light then from equation (7.5), the resultant amplitude reaching at *P* may be written as (*n* is odd)

$$u_p = u_1 + u_3 + u_5 + \ldots\ldots\ldots + u_n \qquad \text{....... (7.13)}$$

In this case if all the zones are transparent to light then from equation (7.5), the resultant amplitude at *P* is given by

$$u_p = u_1 - u_2 + u_3 - u_4 + \ldots\ldots\ldots + u_n \qquad \text{....... (7.14)}$$

For large value of *n*, from equation (7.10), we have,

$$u_p = \frac{u_1}{2} \qquad \text{....... (7.15)}$$

If we compare the values of the resultant amplitudes from equations (7.13) and (7.15), we find that, when the even numbered zones are opaque the intensity at point *P* is much greater than that when all the zones are transparent to incident light. Again from the above discussion we can state that a zone plate behaves like a converging lens. The focal length of the zone plate may be given by

$$f_n = b = \frac{r_n^2}{n\lambda} \qquad \text{....... (7.16)}$$

Therefore, the focal length of a zone plate varies with the wavelength of incident light that is why it is called a multi foci zone plate. For this reason if white light is made to incident on a zone plate different colours come to focus on screen at different points and it shows chromatic aberration.

7.8.2. Action of a Zone Plate

Refer to figure 7.14; AB is the section of zone plate perpendicular to the plane of paper, *S* is the point light source at a distance *a* from zone plate and point *P* is on the screen placed at a distance *b* from the zone plate. As compared to the radii of zones, the distance of source from the zone plate is extremely large and therefore we can take approximation as $SO \approx SM_1 \approx SM_2 \ldots\ldots = a$. The position of the screen is chosen such that the light rays reaching at *P* from successive zones have a path difference of $\lambda/2$. We can write

$$SO + OP = a + b \qquad \ldots\ldots. \ (7.17)$$

$$SM_1 + M_1P \approx SO + (OP + \lambda/2) = a + b + \lambda/2 \qquad \ldots\ldots. \ (7.18)$$

Similarly, $$SM_2 + M_2P = a + b + 2\,\lambda/2 \qquad \ldots\ldots. \ (7.19)$$

Now from right angle triangle ΔSOM_1, we have,

$$(SM_1)^2 = (SO)^2 + (M_1O)^2 \quad \text{or} \quad SM_1 = (a^2 + r_1^2)^{1/2} = a\left(1+\frac{r_1^2}{a^2}\right)^{1/2}$$

Since $a \gg r_1$, expanding above and neglecting higher order terms, we get,

$$SM_1 = a\left(1+\frac{r_1^2}{2a^2}\right)=\left(a+\frac{r_1^2}{2a}\right) \qquad \ldots\ldots. \ (7.20)$$

Proceeding in a similar way we can obtain,

$$M_1P = \left(b+\frac{r_1^2}{2b}\right) \qquad \ldots\ldots. \ (7.21)$$

Substituting values of SM_1 and M_1P from equations (7.20) and (7.21) in the left hand side of equation (7.18), we get,

$$\left(a+\frac{r_1^2}{2a}\right)+\left(b+\frac{r_1^2}{2b}\right) = a + b + \lambda/2$$

or $$r_1^2\left(\frac{1}{a}+\frac{1}{b}\right) = \lambda$$

From equation (7.19), we have, $$r_2^2\left(\frac{1}{a}+\frac{1}{b}\right) = 2\lambda$$

Proceeding similarly for higher order zones, we obtain

$$r_n^2\left(\frac{1}{a}+\frac{1}{b}\right) = n\lambda \qquad \ldots\ldots. \ (7.22)$$

Now comparing the zone plate with converging device like convex lens and using similar sign convention for the distances of the object and image from the lens, the equation (7.22) may be modified as

$$\left(\frac{1}{b}-\frac{1}{a}\right)=\frac{n\lambda}{r_n^2} \qquad \text{....... (7.23)}$$

This equation is similar to the lens equation $\left(\frac{1}{v}-\frac{1}{u}\right)=\frac{1}{f}$. Thus a zone plate behaves like a converging lens of focal length, $f_n=\frac{r_n^2}{n\lambda}$. Thus the focal length of zone plate depends on the number of zones and the wavelength of light used.

7.8.3. Multiple Foci of Zone Plate

A zone plate has a multiple foci. In order to prove this, taking an object at infinity, i.e. at $a=\infty$ in equation (7.23), we get, $r_n^2=bn\lambda$ and therefore, the area of n^{th} zone is given by

$$A_n=\pi r_n^2-\pi r_{n-1}^2=\pi[n\lambda b-(n-1)\lambda b]=\pi\lambda b \qquad \text{....... (7.24)}$$

Since the object is at infinity, the light rays will be parallel to principal axis and the image will be formed at the principal focus at a distance $b=\frac{r_n^2}{n\lambda}$ from the zone plate.

If we take a point P_3 at a distance $b/3$ from the zone plate somewhere in between O and P then the area of each half period zone with respect to P_3 will now becomes $\pi\lambda(b/3)$, that is, one third to the previous case. Thus each zone, in this case, can be assumed to contain three half period elements corresponding to P_3. If the amplitude due to these elements are represented by m_1, m_2, m_3 etc. then the first zone (amplitude u_1) will consist of the first three elements (amplitudes m_1, m_2 and m_3), second zone (amplitude u_2) will consist of the next three elements (amplitudes m_4, m_5 and m_6) etc. Again similar to half period zones there will be a phase difference of π between the successive elements. Thus while adding the amplitudes; the m_1 will be taken positive, m_2 as negative etc. Substituting the values of u_1, u_2, u_3 etc. with m_1, m_2, m_3 etc., equation (7.13) changes to

$$u_{p_3}=(m_1-m_2+m_3)+(m_7-m_8+m_9)+(m_{13}-m_{14}+m_{15})+\ \dots\dots\dots$$

$$=\left(m_1-\frac{m_1+m_3}{2}+m_3\right)+\left(m_7-\frac{m_7+m_9}{2}+m_9\right)+\left(m_{13}-\frac{m_{13}+m_{15}}{2}+m_{15}\right)+\dots\dots$$

$$=\frac{1}{2}(m_1+m_3+m_7+m_9+m_{13}+m_{15}+\ \dots\dots) \qquad \text{....... (7.25)}$$

Here it should be noted that each of the amplitudes m_1, m_2, m_3 etc is one third of u_1, u_2, u_3 etc.

If we compare the equations (7.13) and (7.25), we find that the intensity reaching at P_3 is sufficiently large but is less than that reaching at P. Thus the image of S is also formed at P_3 and therefore, it may be taken as the second focal point. The second focal length is given by

$$f_3=\frac{r_n^2}{3n\lambda} \qquad \text{....... (7.26)}$$

Similarly the images of S can be formed on points P_5, P_7, P_9 etc. but with decreasing intensity. The distance of these points from the zone plate are $\frac{r_n^2}{5n\lambda}$, $\frac{r_n^2}{7n\lambda}$, $\frac{r_n^2}{9n\lambda}$ etc. Thus a zone plate has multiple foci given by $f_1 = \frac{r_n^2}{n\lambda}$, $f_3 = \frac{r_n^2}{3n\lambda} = \frac{f_1}{3}$, $f_5 = \frac{r_n^2}{5n\lambda} = \frac{f_1}{5}$ etc.

7.8.4. Comparison of Zone Plate and Lens

Some of the features of zone plate are similar to a lens and in some it has dissimilarity. The following are the resemblance and differences between the two.

(i) Similar to a lens, a zone plate forms an image of an object placed on its axis. The same sign convention is used while representing the distance of the object and image in both the cases.

(ii) The focal length formula in terms of distance of object and image for zone plate is $\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{1}{f}$ and for the convex lens is $\left(\frac{1}{v} - \frac{1}{u}\right) = \frac{1}{f}$, which are identical.

(iii)The image due to a convex lens is more intense as compared to that due to a zone plate.

(iv)The convex lens has a focal length given by $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ which depends on wavelength (refractive index varies with wavelength) and the focal length of zone plate $f_n = \frac{r_n^2}{n\lambda}$ also varies with wavelength. Hence both exhibit chromatic aberration.The focal length of a zone plate is inversely proportional to the wavelength hence red rays come to focus at a smaller distance from the zone plate than violet rays. The reverse is true for convex lens. Thus $f_v > f_r$ in zone plate while $f_r > f_v$ in lens. The order of colours in chromatic aberration is therefore opposite in the two cases.

(v) A convex lens has one focal length for a fixed wavelength while a zone plate has a number of foci at which the images of diminishing intensities are formed.

Example 7.5: Calculate the focal length of the zone plate and the radius of the first zone when a point source of light of wavelength 6x10^{-7} m is placed at a distance of 100 cm from a zone plate. Its image is formed at a distance of 200 cm on the other side.

Solution: For a zone plate we have. $\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{n\lambda}{r_n^2} = \frac{1}{f}$. Given that, a = 1 m, b = 2 m and λ = 6x10^{-7} m. Thus $\frac{1}{f} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$ or $f = \frac{2}{3}\ m$.

For first zone, $f = \frac{{r_1}^2}{1\times\lambda}$, thus $r_1^2 = f \times \lambda = \frac{2}{3} \times 6\text{x}10^{-7}$ or r_1 = 6.32x10^{-4} m.

Example 7.6: A plane wavefront of light of wavelength 5x10^{-5} cm fall on a zone plate. The radius of the first half period zone is 0.5 mm. Where should a screen be placed so that the light is focused at the brightest spot?

Solution: We know that the brightest spot is formed at the first focus of the plate, i.e. at f_1. Given that $r_1 = 0.5\ mm = 5x10^{-2}\ cm$ and $\lambda = 5 \times 10^{-5}\ cm$

$$f_n = \frac{r_n^2}{n\lambda}, \text{ Therefore, } f_1 = \frac{{r_1}^2}{\lambda} = \frac{\left(5\times10^{-2}\right)^2}{5\times10^{-5}} = 50\ cm$$

Example 7.7: Calculate the radius of 10th zone in a zone plate of focal length 0.2 m for light of wavelength $5\text{x}10^{-7}$ m.

Solution: From $f_n = \frac{r_n^2}{n\lambda}$, we have, $0.2 = \frac{{r_{10}}^2}{10\times5\times10^{-7}}$ or $r_{10} = 0.01\ m = 1.0\ cm$

Example 7.8: Calculate the radii of first three clear elements of a zone plate which is designed to bring a parallel light of wavelength 6000 Å to its first focus at a distance of two meters.

Solution: It is Given that, $f = b = 2.0\ m$, $\lambda = 6000\text{Å} = 6\times10^{-7}\ m$.

If odd number half period zones are clear (transparent) then taking n=1, 3, 5 in the expression $r_n = \sqrt{nb\lambda}$, we get $r_1 = \sqrt{f\lambda} = \sqrt{6\times10^{-7}\times2} = 10.95\times10^{-4}$ m.

$r_3 = \sqrt{3f\lambda} = \sqrt{3\times6\times10^{-7}\times2} = 1.9\times10^{-3}$ m.

$r_5 = \sqrt{5f\lambda} = \sqrt{5\times6\times10^{-7}\times2} = 2.45\times10^{-3}$ m.

SAQ 7: What is the radius of first zone in a zone plate of primary focal length 20 cm for a light of wavelength 5000 Å.

SAQ 8: If the focal length of zone plate is 1 m for light of wavelength $6.0\text{x}10^{-7}$ m. What will be its focal length for the wavelength $5\text{x}10^{-7}$m.

7.9. DIFFRACTION AT A STRAIGHT EDGE

To show the diffraction effect of a straight edge, the light from a monochromatic light source *S* is passed through a narrow slit *AB* and a sharp edge of an opaque obstacle like blade is placed in its path as shown in figure 7.15. The slit, opaque obstacle and screen $P'P''$ are parallel to each other and perpendicular to the plane of the paper. The sharp edge is placed in such a way that the line joining the slit to edge *O* when reproduced meet the screen at *P* and *OP* is normal to screen.

In the absence of diffraction of light due to sharp straight edge there should be a uniform illumination above point *P* and complete darkness below it. As we move towards *P'*, unequally spaced bright and dark bands are obtained near *P*. On further moving towards *P'*, i.e. with increasing value of *x* the intensity reaches a steady value I_o resulting a uniform illumination. Because of the diffraction effect, the light enters to a certain distance below *P* (towards *P''*) in the geometrical shadow region.

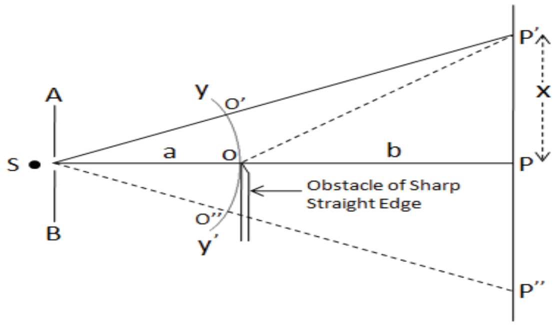

Figure 7.15

In this region the intensity of light decreases to zero very rapidly without forming maxima and minima in a small but finite distance as shown in intensity distribution curve of figure 7.16. If the average intensity is I_o then at point *P* on the screen (corresponding to the edge) it reduces to $I_o/4$. This all is due to the diffraction of light produced by sharp straight edge.

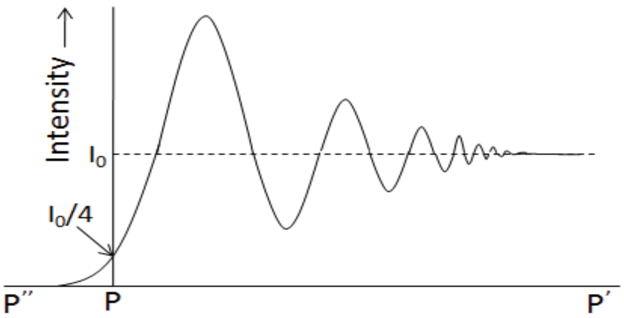

Figure 7.16

7.9.1. Theoretical Analysis

Refer to figure 7.15. Suppose we want to find the resultant at any point, say *P'*, on the screen. The pole of the wavefront *YY'* with respect to point *P'* will be *O'*. With *P'* as centre if we draw the circles of radii $O'P'+\lambda/2$, $O'P'+2\lambda/2$, $O'P'+3\lambda/2$ etc, the wavefront is divided into half period strips. Thus for point *P*', the wavefront is divided in two similar parts; one above point *O'* another below it. The light from entire upper half portion of the wavefront reaches to *P'*. The resultant due to this will be equivalent to one half to that due to first half period strip, i.e. $m_1/2$. Now the number of half period strips within the lower half portion of the wavefront, i.e. *O'O* will depend on the position of the point *P'* on the screen. Suppose the lower half portion contains only one half period strip then the amplitude due to it at *P'* will be only m_1 and therefore, the total amplitude at *P'* by whole of the exposed wavefront is given by $\frac{m_1}{2}+m_1$. This is the position of first maximum.

If *O'O* contains two, three, four etc half period strips then the resultant amplitude at *p'* is given by $\frac{m_1}{2}+m_1-m_2$, $\frac{m_1}{2}+m_1-m_2+m_3$, $\frac{m_1}{2}+m_1-m_2+m_3-m_4$ etc. and the position of *P'* gives the position of first minimum, position of second maximum and the position of second minimum respectively. Thus at point *P'*, a maximum or a minimum is formed according as *O'O* contains odd or even number of half period strips.

As we move away from *P* towards *P'* alternate maxima and minima are obtained. From the previous discussion we see that the amplitude or intensity of these maxima and minima are comparable, hence the bands have a poor contrast. If the point of consideration is at a sufficiently large distance from *P* then entire upper half and a large number of half period strips of the lower half are exposed. The diffraction bands merge together to produce uniform illumination. The resultant amplitude at the point of consideration, in this case, is therefore, $\frac{m_1}{2}+\frac{m_1}{2} = m_1$ and the intensity is $m_1{}^2$.

7.9.2. Positions of Maximum and Minimum Intensities

In figure 7.15, the path difference between the rays $O'P'$ and OP' is given by

$\Delta = OP'$- $O'P'$= $(OP^2 + PP'^2)^{1/2} - (SP' - SO')$=$(OP^2 + PP'^2)^{1/2} - \left[\{SP^2 + PP'^2\}^{1/2} - SO'\right]$

$$= (b^2 + x^2)^{1/2} - \left[\{(a+b)^2 + x^2\}^{1/2} - a\right]$$

∵ Y*Y'* is the spherical wavefront of the point light source *S* with *S* as a centre, thus *SO'* = *SO* = a, is the radius of the sphere.

In actual experimental set up we have, *x<<b*. Thus taking *b* out (common) from the first term and *(a+b)* out from the second term on the right hand side of the above equation, expanding the series and neglecting higher order terms, we obtain

$$\Delta = b\left\{1 + \frac{x^2}{2b^2}\right\} - (a+b)\left\{1 + \frac{x^2}{2(a+b)^2}\right\} + a = \frac{x^2}{2}\cdot\frac{a}{b(a+b)} \quad .... (7.27)$$

Now if *O'O* contains an odd number of half period strips then a maximum will be formed at point *P'* and the path difference Δ, in this case, will be an odd number of half-wavelengths, and vice-versa. Thus for maxima we have,

$$\Delta = (2n-1)\frac{\lambda}{2} \quad .... (7.28)$$

For minima we have, $\Delta = 2n.\frac{\lambda}{2}$ .... (7.29)

On comparing equations (7.27) and (7.28), we get the position of n^{th} maximum as

$$x_n = \sqrt{\frac{(2n-1)(a+b)b\lambda}{a}} = K\sqrt{2n-1} \quad .... (7.30)$$

Where, $K = \sqrt{\frac{(a+b)b\lambda}{a}}$, is a constant.

Similarly the comparison of equations (7.27) and (7.29) gives the position of n^{th} minimum as

$$x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}} = K\sqrt{2n} \qquad \text{....... (7.31)}$$

From equation (7.30), we have, $x_1 = K,\ x_2 = K\sqrt{3}, x_3 = K\sqrt{5}$ etc. Thus the separations between successive maxima are $x_2 - x_1 = 0.732K, x_3 - x_2 = 0.504K, x_4 - x_3 = 0.409$ etc. We see that with increasing order of maxima the separation between consecutive maxima decreases and the fringes come closer. The same is true for minima.

7.9.3. Intensities at Various Positions

The intensity variation curve is shown in figure 7.16. Now we will find out the value of intensity at some specific points.

(i) Intensity at the Edge of Geometrical Shadow

In figure 7.16 the edge of geometrical shadow is represented by *P*. The pole of this edge at wavefront is point *O*, which is nothing but the edge of sharp obstacle. Thus with respect to the edge of geometrical shadow region (point *P*), the incident wavefront can be divided in two parts; one above point *O* (*OY)* and other below point *O* (*OY'*). The light from the entire upper half portion of the wavefront reaches to point *P* while the light from the lower half portion of the wavefront is completely cut off by sharp edge obstacle. The resultant amplitude at *P*, in this case, is $m_P = m_1 - m_2 + m_3 + m_4 - \ldots\ldots\ldots$, *which is* $m_1/2$. Thus the resultant intensity at *P* is $m_1^2/4 = I_o/4$. Where I_o is the value of intensity at *P* in the absence of obstacle.

(ii) Intensity at a Point Inside the Geometrical Shadow

If the point of consideration is inside the geometrical shadow region then the pole of the point will be below point *O,* i.e. in the wavefront region *OY'*. Suppose we take a point *P''* then its pole will be *O''*. In this case the complete lower half portion and most of the upper half portion of the wavefront is obstructed by the obstacle. Only a small part of the upper half portion of the wavefront (*OY*) is exposed. As we move down gradually from point *P* inside geometrical shadow, the first, the first two, the first three etc. half period strips of the upper half of the wavefront are obstructed and the amplitudes are thus $m_2/2$, $m_3/2$, $m_4/2$ etc. respectively. The intensities, therefore, will be $(m_2/2)^2$, $(m_3/2)^2$, $(m_4/2)^2$ etc. respectively.

Since the amplitudes m_1, m_2, m_3 etc. are in decreasing order of magnitude, the intensity of light decreases rapidly as we move inside the geometrical shadow. This is because of the fact that most of the effective half period strips of the upper half portion of wavefront are cut off.

Example 7.9: A narrow slit illuminated by light of wavelength 4900 Å is placed at a distance of 3m from a straight edge. If the distance between the straight edge and screen is 6 m, calculate the distance between the first and fourth band.

Solution: For minima we have, $x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}} = K\sqrt{2n}$, where $K = \sqrt{\frac{(a+b)b\lambda}{a}}$

It is given that b = 6 m, a = 3 m, λ = 4.9x10^{-7} m.

Therefore, $K = \sqrt{\frac{(3+6)\times 6\times 4.9\text{x}10^{-7}}{3}} = 2.97x10^{-3}$

For first minimum, $x_1 = K\sqrt{2} = 2.97\times 10^{-3}\times\sqrt{2} = 4.20\times 10^{-3}m$

For fourth minimum, $x_4 = K\sqrt{8} = 2.97\times 10^{-3}\times\sqrt{8} = 8.40\times 10^{-3}$

Separation between the two, $x_4 - x_1 = (8.40 - 4.20)\ x\ 10^{-3} = 4.20\times 10^{-3}m$

SAQ 9: In an experiment with straight edge diffraction, the slit to edge distance is 1.0 meter and the edge to screen distance is 2.0 m. If $\lambda = 6000$ Å, calculate the position of the first three maxima and their separation.

7.10. SUMMARY

In this unit you have studied that Huygens's principal is the basic principle to explain the diffraction phenomenon. Diffraction is mainly due to interference of the secondary wavelets. Diffraction pattern is formed whenever a wave encounters an object or aperture, the size of which is comparable to wavelength of light. To make the concept more clear the difference between interference and diffraction, construction and theory of half period zones and zone plate are explained. It is stated that for $b>>\lambda$ the radii of half period zones are proportional to square root of natural numbers and the zones have the same areas. The expressions for radius and area are given by $\sqrt{n\lambda b}$ and $\pi\lambda b$. If the incident wavefront contains a large number of half period zones and all zones are exposed then the resultant amplitude at a point on the screen will be equal to half of that due to first zone, i.e. $u_1/2$. With the help of zone theory it is proved that the light propagates along a rectilinear path. The zone plate may be used as a focusing device and the focal length of it is given by the expression $\frac{1}{f_n} = \left(\frac{1}{b} - \frac{1}{a}\right) = \frac{n\lambda}{r_n^2}$. It is a multiple foci device having focal lengths $\frac{r_n^2}{n\lambda}, \frac{r_n^2}{3n\lambda}, \frac{r_n^2}{5n\lambda}$ etc. In some of the features, the zone plate, resembles with a lens and has some dissimilarity.

The formation of diffraction pattern is explained by taking the obstacle in the form of a sharp and straight edge. If almost all the wavefront is exposed, the amplitude produced at a point on the screen is m_1 and the intensity is m_1^2. The maxima and minima formed are not equally spaced. Their position of maxima is given by $x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}} = K\sqrt{2n}$ and that of minima is given by $x_n = \sqrt{\frac{(2n-1)(a+b)b\lambda}{a}} = K\sqrt{2n-1}$. If I_o is the value of intensity at a point on the screen in the absence of obstacle then $I_o/4$ will be the intensity at the edge of geometrical shadow. If we move inside the geometrical shadow region the intensity decreases and diminishes to zero rapidly.

7.11 GLOSSARY

Annular – ring-shaped, forming a ring.

Aperture – an opening, a gap or a space through which light passes in an optical or photographic instrument.

Ascribe – attribute or impute, regard as belonging.

Attribute – ascribe to or regard as the effect of (a stated cause).

Convention – general agreement, esp. on social behaviour etc. by implicit consent of the majority, a custom or customary practice esp. an artificial or formal one.

Converse – opposite, contrary, reverse.

Depict – to describe.

Distinct – not identical, separate, individual, different in kind or quality, unlike.

Emanate – issue, originate (from a source), proceed.

Evident – plain or obvious (visually or intellectually), manifest.

Illumination – an act to light up or to make bright.

Inflexion – the act or condition of inflecting or being inflected, an instance of this.

Lateral – of, at, towards, or from the side or sides, in direct line.

Monochromatic – light or other radiation of single wavelength, containing only one colour.

Obstruct – block up, make hard or impossible to pass along or through.

Opaque – not transmitting light, impenetrable to light.

Rectilinear – bounded or characterized by straight lines, in or forming a straight line.

Render – cause to be or become, make.

Respectively – in the order mentioned, for each separately or in turn.

Reveal – display or show, allow to appear, disclose, divulge, betray.

Vary – undergo change (become or be different).

7.12 TERMINAL QUESTIONS

1. Calculate the radii and areas of the first two half period zones for a plane wavefront. The point of observation is at a distance of 1.0 m from theb wavefront and wavelength of light is 4900 Å.

2. The diameter of the first ring of a zone plate is 1.1 mm. If plane waves (6000 Å) fall on the plate, where should the screen be placed so that light is focused to a brightest spot?

3. A light of wavelength 5000 Å is allowed to fall on a zone plate for which the radius of the first zone is 3×10^{-2} cm. Find the first three focal lengths for this zone plate.

4. Light of wavelength 5896 Å is made to incident on a zone plate placed at a distance of 150 cm from it. The image of the point source is obtained at a distance of 3 m on the other side. What will be the power of equivalent lens which may replace the zone plate withought disturbing the set up? Also calculate the radius of the first zone of the plate.

5. For axial point source for a zone plate, a series of images is obtained. If the sharpest image is obtained at 30 cm and the next sharpest at 6 cm on the other side of the source, calculate the distance of the source from the zone plate.

6. For a light of wavelength 4000Å, the brightest image is formed by a zone plate at a distance of 20 cm for an object placed at a distance of 20 cm from it. Calculate the number of Fresnel's zones in a radius of 1 cm of that plate.

7. A point source of $\lambda = 5.5 \times 10^{-7} m$ is placed 2 meters away along the axis of a circular aperture of radius 2 mm. On the other side a screen is moved along the axis from infinity to closer distances. Calculate the first three positions where minima are observed.

8. A parallel beam of wavelength 6×10^{-7} m falls normally on a narrow circular aperture of radius 0.9 mm. At what distance along the axis will the first maximum intensity be observed?

9. A straight edge is placed at a distance of 50 cm from a slit illuminated by monochromatic light of wavelength 5000 Å. If the distance of the screen from the edge is 1.50 m, calculate the positions of first, second, third and tenth bright fringe from the edge of the geometrical shadow. Also find the separation between first-second and second-third bright fringes.

7.13 OBJECTIVE TYPE QUESTIONS

Q1. The bending of light rays round the corners of an obstacle is called

(a) interference (b) polarization

(c) dispersion (d) diffraction

Q2. For obtaining the diffraction pattern the size of the obstacle should be

(a) 10 mm (b) 10^{-1} mm

(c) 10^{-4} mm (d) 0.1 cm

Q3. The phenomenon of diffraction was discovered by

(a) Francesco Maria Grimaldi (b) Isaac Newton

(c) Fraunhofer (d) Huygen

Q4. The tip of a needle does not give a sharp image on the screen because of the following

(a) reflection (b) diffraction

(c) polarization (d) refraction

Q5. Fresnel half period zones differ from each other by a phase difference of

(a) 2π (b) π

(c) $\pi/2$ (d) $\pi/4$

Q6. For a light of wavelength $5x10^{-7}$m, a zone plate of focal length 0.5 m is to be constructed. The radius of first zone will be

(a) 0.25 cm (b) $2.5x10^{-2}$ cm

(c) 0.5 cm (d) $5x10^{-2}$ cm

Q7. The constant area of half period zone is given by

(a) $\pi b\lambda$ (b) $\pi b/\lambda$

(c) $\lambda/\pi b$ (d) $2\lambda/\pi b$

Q8. The first (principal) focal length of a zone plate has least value for the following colour

(a) red colour (b) green colour

(c) violet colour (d) yellow colour

Q9. The focal length of a zone plate is given by the expression

(a) $\frac{r_n}{n\lambda}$ (b) $\frac{r_n^2}{n}\lambda$

(c) $\frac{r_n^2}{n\lambda}$ (d) $\frac{r_n^2}{\lambda}n$

Q10. A zone plate behaves like

(a) concave lens (b) convex lens

(c) plane mirror (d) glass plate

7.14 ANSWERS/HINTS

7.14.1 Self Assessment Questions

1. Refer article 7.3, **2.** Refer article 7.4, **3.** Refer article 7.5,

4. It is given that the radius of n^{th} zone is given by $r_n = \sqrt{nb\lambda} = 1.0\ cm = 10^{-2} m$ and the area of zone, $A_n = \pi b\lambda = 3.14 \times 10^{-7}\ m^2$

Thus, $\frac{r_n^2}{A_n} = \frac{nb\lambda}{\pi b\lambda} = \frac{n}{\pi}$ or $n = \pi\,\frac{r_n^2}{A_n} = 3.14 \times \frac{(10^{-2})^2}{3.14\times10^{-7}} = 1000$

5: It is given that $\lambda = 5\times10^{-7}$ m, b = 1 m and n = 2. If A_n is the area of hole of radius r_n containing n-half period zones each of area $\pi b\lambda$ then, we have, $A_n = \pi r_n^2 = n.\ \pi b\lambda$

Substituting the given values in the above equation, we get,

$\pi r_n^2 = 2 \times \pi \times 1 \times 5 \times 10^{-7}$ or $r_n = 10^{-3}$ m, thus diameter, $d_n = 2\times10^{-3}$ m.

6: The intensity due to whole wavefront is only one fourth to that due to first half period zone, therefore, Radius of opening = Radius of first half period zone = $\sqrt{b\lambda} = 4.47\text{x}10^{-2}$ cm, b = 40 cm (given).

Thus, $\lambda = \frac{(4.47\text{x}10^{-2})^2}{b} = \frac{(4.47\text{x}10^{-2})^2}{40} = 5 \times 10^{-5}\ cm$

7: Hint: $f_n = \frac{r_n^2}{n\lambda}$, $\therefore\ r_1 = \sqrt{f\lambda} = 3.16 \times 10^{-4}\ m.$

8: Hint: If *f* and *f'* are the focal lengths for the wavelengths λ and λ' then we have

$f = \frac{r_n^2}{n\lambda}$ and $f' = \frac{r_n^2}{n\lambda'}$. Dividing we get, $f' = f\,\frac{\lambda}{\lambda'} = 1 \times \frac{6\times10^{-7}}{5\times10^{-7}} = 1.2\ m$

9: For maxima we have, $x_n = \sqrt{\frac{(2n-1)(a+b)b\lambda}{a}} = K\sqrt{2n-1}$, where $K = \sqrt{\frac{(a+b)b\lambda}{a}}$

It is given that b = 2 m, a = 1 m, $\lambda = 6000$ Å = 6×10^{-7} m.

Therefore, $K = \sqrt{\frac{(1+2)\times 2\times 6\text{x}10^{-7}}{1}} = 1.897x10^{-3}m$

For first maximum, $x_1 = K\sqrt{1} = 1.897 \times 10^{-3} \times \sqrt{1} = 1.897 \times 10^{-3}m$

For second maximum, $x_2 = K\sqrt{3} = 1.897 \times 10^{-3} \times \sqrt{3} = 3.286 \times 10^{-3}m$

For third maximum, $x_3 = K\sqrt{5} = 1.897 \times 10^{-3} \times \sqrt{5} = 4.243 \times 10^{-3}m$

Separation between the two, $x_4 - x_1 = (8.40 - 4.20)\ x\ 10^{-3} = 4.20 \times 10^{-3}m$

7.14.2 Terminal Questions

1. Radii are $7\text{x}10^{-4}$m and $9.9\text{x}10^{-4}$m respectively, and area of each is $1.54\text{x}10^{-6}\text{m}^2$, **2.** 50 cm, **3.** 18 cm, 6 cm, 3.6 cm, **4.** 1.0 dioptre, 0.0768 cm, (**Hint:** Power, $P = \frac{1}{f}$ dioptre where $\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$ and $r_n = \sqrt{fn\lambda}$, **5.** a=30 cm (**Hint:** $\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{n\lambda}{r_n^2}$, Thus $\left(\frac{1}{a} + \frac{1}{30}\right) = \frac{n\lambda}{r_n^2}$ and $\left(\frac{1}{a} + \frac{1}{6}\right) = \frac{3n\lambda}{r_n^2}$), **6.** 2500 (**Hint:** $n = \frac{r_n^2}{f\lambda}$ where f can be calculated by $\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{1}{f}$), **7.** 19.98 m, 3.076 m, 1.664 m (**Hint:** $n = \frac{r^2}{\lambda}\left(\frac{1}{a} + \frac{1}{b}\right)$, $\therefore n = 3.636 + \frac{7.273}{b}$, For first three positions of minima, *n=4, 6, 8)*, **8.** 1.35 m (**Hint:** For parallel beam, $a=\infty$, and for first maximum *n=1*), **9.** $x_1 = 0.173$ cm, $x_2 = 0.300$ cm, $x_3 = 0.66$ cm, $x_{10} = 0.533$ cm, $x_2\text{-}x_1 = \beta_{12} = 0.127$ cm, $\beta_{23} = 0.066$ cm,

7.14.3 Objective Type Questions

1. (b), **2.** (c), **3.** (a), **4.** (b), **5.** (b), **6.** (d), **7.** (a), **8.** (a), **9.** (c), **10.** (b)

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UNIT 8: FRAUNHOFER DIFFRACTION

CONTANTS

8.1 INTRODUCTION

If an opaque obstacle is placed between a source of light and a screen then light bends around the corner of the obstacle into the geometrical shadow. This bending of light is called diffraction. The phenomenon of diffraction depends on the size of the obstacle and the wavelength of the light beam.

Diffraction is one particular type of wave interference, caused by the partial obstruction or lateral restriction of a wave. Not all interferences are diffraction; for example, sound waves emitted by two stereo speakers will interfere with each other if they are of the same frequency and have a definite phase relationship, but this is not diffraction. Diffraction will not occur if the wave is not coherent, and diffraction effects become weaker (and ultimately undetectable) as the size of obstruction is made larger and larger compared to the wavelength. In well-defined cases, a diffraction pattern may be observed. It is necessary to mention here that diffraction is not the same as refraction, although both are phenomena in which a wave does not propagate in a single direction.

8.2 OBJECTIVES

After studying this unit, you will be able to

- have the basic idea of diffraction and its various classes.
- know the diffraction output at various structure like single, double and multiple slit.
- introduce the plane diffraction grating.
- determine the missing orders for diffraction spectra.

8.3 CLASSES OF DIFFRACTION

Based on the distance between source, aperture and screen, and also on the shape of wavefront, diffraction pattern is classified into two classes

1. **Fresnel Diffraction-**If the source of light and the screen are at finite distances from the diffracting aperture, then the wavefront falling on the aperture will not be plane (spherical or cylindrical). The diffraction obtained under this type of arrangement is called Fresnel Diffraction. This type of diffraction is also called near-field diffraction. No lenses are used to make the rays parallel or convergent.

   Fresnel Diffraction is obtained when light suffers diffraction at a straight edge, a thin wire, a narrow slit etc. Both the size and shape of the pattern depends on the distance between the diffracting aperture and the screen.

2. **Fraunhofer Diffraction-**If both the source of light and the screen are effectively far enough from the aperture so that the wavefronts reaching the aperture and the screen can be considered plane. Then the source and the screen are said to be at infinite distances from the aperture. This kind of diffraction is called Fraunhofer Diffraction. This is also called far-field diffraction.

Fraunhofer Diffraction is encountered in the case of gratings that contain number of slits. When the screen is moved, the size of the diffraction pattern changes uniformly while the shape of the pattern does not change.

8.4 FRAUNHOFER DIFFRACTION DUE TO A SINGLE SLIT

Let AB is a slit of width b, the diffracted beam through the slit is tilted at an angle θ with respect to straight direction.

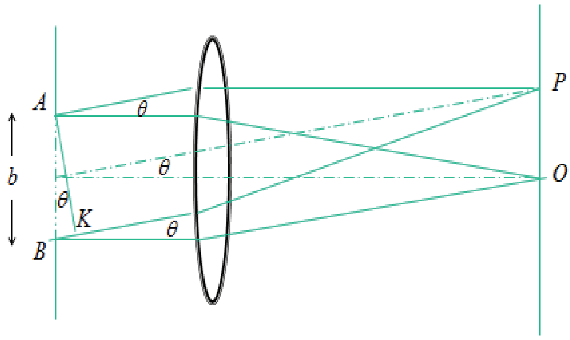

Figure 8.1

Path difference between two rays diffracted from two extreme points of slit

$$= BK = AB\ sin\theta = b\ sin\theta$$

Phase difference $= \frac{2\pi}{\lambda}$ x path difference $= \frac{2\pi}{\lambda}(b\sin\theta)$

Let the width AB of the slit be divide into n equal parts. The amplitude of vibration at P due to the waves from each part will be same, say *a*. The phase difference between the waves from any two consecutive parts is

$$\frac{1}{n}\left(\frac{2\pi}{\lambda}b\sin\theta\right) = 2\beta \text{, say}$$

Then the resultant amplitude at P is given by

$$R = \frac{a\sin(nd/2)}{\sin(d/2)} = \frac{a\sin\left(\frac{\pi b\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi b\sin\theta}{n\lambda}\right)}$$

Let us put $\left(\frac{\pi}{\lambda}b\sin\theta\right) = \alpha$

Then $R = \frac{a\sin\alpha}{\sin(\alpha/n)} = \frac{a\sin\alpha}{\alpha/n} = \frac{na\sin\alpha}{\alpha}$ .... (8.1)

When $n \to \infty$, $a \to 0$, but the product *na* remains finite.

Let $na = A$

The resultant intensity at P, being proportional to the square of the amplitude, is

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{....... (8.2)}$$

Condition for Maxima

$$R = \frac{A\sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$R = \frac{A\sin\alpha}{\alpha} = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right] \qquad \text{....... (8.3)}$$

For $\alpha = 0$, $R = A$

This is the intensity of central maximum

$$\alpha = \left(\frac{\pi}{\lambda} b\sin\theta\right) = 0 \text{ or } \sin\theta = 0$$

Condition for Minima

$$\frac{\sin\alpha}{\alpha} = 0 \text{ or } \sin\alpha = 0 \text{, but } \alpha \neq 0$$

$\alpha = \pm m\pi$, Where m has an integral value 1, 2, 3 except zero

So $$\left(\frac{\pi}{\lambda} b\sin\theta\right) = \pm m\pi \Rightarrow b\sin\theta = \pm m\lambda \qquad \text{....... (8.4)}$$

This equation gives the position of first, second, third etc. minima for m = 1, 2, 3 etc

Secondary Maxima

$$\frac{dI}{d\alpha} = 0$$

or $$\frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

or $$A^2\left(\frac{2\sin\alpha}{\alpha}\right)\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha = \tan\alpha = y \ (say)$$

$$y = \alpha \text{ and } y = \tan\alpha$$

The maxima will occur when

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

or $$\alpha = (2n+1)\frac{\pi}{2} \quad n = 1,2,3..... \qquad .... (8.5)$$

These are points of secondary maxima

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad .... (8.6)$$

Put $$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}\ etc.$$

$$I_1 = \frac{4}{9\pi^2}I_0,\ I_2 = \frac{4}{25\pi^2}I_0,\ I_3 = \frac{4}{49\pi^2}I_0\ etc$$

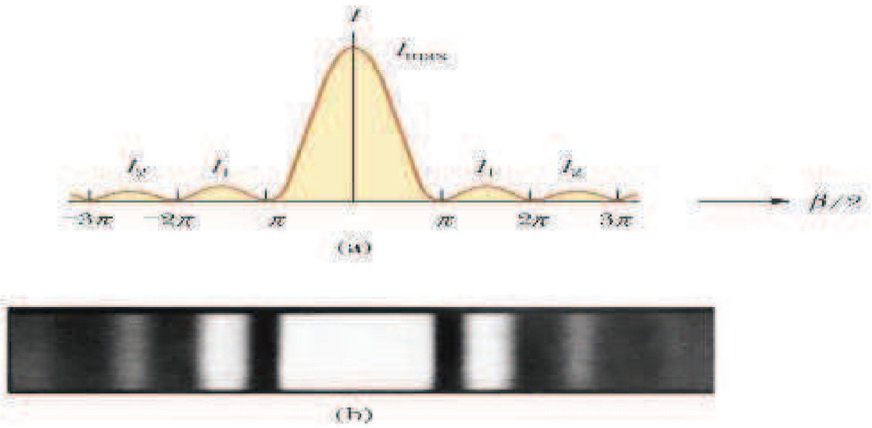

Figure 8.2

8.5 FRAUNHOFER DIFFRACTION DUE TO DOUBLE SLIT

Let a parallel beam of monochromatic light of wavelength λ be incident normally upon two parallel slits AB and CD, each of width *b* and their separation as *d*. The distance between the corresponding points of two slits will be *(b+d)*.

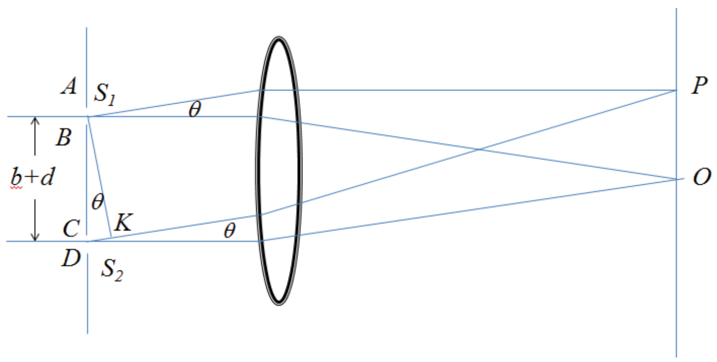

Figure 8.3

Suppose each slit diffracts the beam in a direction making an angle θ with the direction of incident beam. From the theory of diffraction at single slit, the resultant amplitude will be

$$\frac{A\ Sin\ \alpha}{\alpha}$$

Where $$\alpha = \frac{\pi b Sin\ \theta}{\lambda}$$

Now consider the two slits equivalent to two coherent sources, placed at the middle points S_1 and $S_2$of the slits and each sending a wavelet of amplitude $\frac{A\ Sin\ \alpha}{\alpha}$.

Therefore, the resultant amplitude at point P on the screen will be the result of the interference between two waves of same amplitude $\frac{A\ Sin\ \alpha}{\alpha}$ and having a phase difference δ.

∴ Path difference between the wavelets coming from S_1 and S_2in direction θ is given by

$$S_2K = (b+d)\ sin\theta$$

Phase difference = $\frac{2\pi}{\lambda}$ x path difference = $\frac{2\pi}{\lambda}(b+d)\sin\theta) = 2\beta$

Resultant amplitude R at point P can be obtained by vector addition method as

$$I = R^2 = 4A^2 \frac{Sin^2\alpha\ Cos^2\beta}{\alpha^2} \qquad \text{....... (8.7)}$$

Here $\frac{Sin^2\alpha}{\alpha^2}$ gives the diffraction pattern due to each individual slit and $Cos^2\beta$ gives the interference pattern due to double slit. $\frac{Sin^2\alpha}{\alpha^2}$ gives a central maximum in the direction β= 0, having alternate minima and secondary maxima of decreasing intensity on either side.

The minima are obtained in the directions given by

$$Sin\alpha = 0 \qquad \text{or} \qquad \alpha = \pm m\pi$$

∵ $$\alpha = \frac{\pi b\ Sin\ \theta}{\lambda}$$

∴ $$b\,Sin\theta = \pm m\pi \qquad \text{....... (8.8)}$$

Where m = 1, 2, 3…. (except zero).

The term $Cos^2\beta$ in the intensity pattern gives a set of equidistant dark and bright fringes.

$$Cos^2\beta = 1$$

∴ $$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(b+d)\sin\theta = \pm n\pi$$

$$(b+d)\sin\theta = \pm n\lambda \qquad \text{....... (8.9)}$$

Where n = 0, 1, 2, 3…., correspond to zero-, first-, second- etc. order Maxima.

8.5.1 Missing Orders

In the output intensity pattern of a double slit, for certain values of d, few interference maxima become absent.

As, the directions of interference maxima are given by

$$(b + d)\sin\theta = n\lambda \qquad \text{....... (8.10)}$$

The directions of diffraction minima are given by

$$b\sin\theta = m\lambda \qquad \text{....... (8.11)}$$

If the values of *b* and *d* are such that both the equations are satisfied for the same value of a, then a certain interference maximum will overlap the diffraction minimum and hence the spectrum order will be missing (absent).

Dividing equation (8.10) by equation (8.11), we get,

$$\frac{b+d}{b} = \frac{n}{m} \qquad \text{....... (8.12)}$$

If *b=d*

$$\frac{n}{m} = 2 \quad or\ n = 2m.\ If\ m = 1,2,3.......etc.,\ then\ n = 2,4,6.......etc$$

This means that the 2, 4, 6 etc. orders of interference maxima will be missing in the diffraction pattern. Thus the central diffraction maxima will have three interference maxima (the zero order and two first-orders).

If d=2b

$$\frac{b+2b}{b} = \frac{n}{m} \quad or\ n = 3m.\ If\ m = 1,2,3.......etc., \quad n = 3,6,9.......etc$$

This means that 3rd, 6th, 9th etc, orders of interference maxima will be missing in the diffraction pattern. On both sides of the central maximum, the number of interference maximum is 2 and hence there will be five interference maxima in the central diffraction maximum.

8.6 FRAUNHOFER DIFFRACTION AT CIRCULAR APERTURE

The problem of diffraction at a circular aperture was first solved by Airy in 1835. The amplitude distribution for diffraction due to a circular aperture forms an intensity pattern with a bright central band surrounded by concentric circular bands of rapidly decreasing intensity (Airy pattern). The 1st maximum is roughly 1.75% of the central intensity. 84% of the light arrives within the central peak called the airy disk

Let us consider a circular aperture of diameter *d* is shown as AB in figure below. A plane wave front WW' is incident normally on this aperture. Every point on the plane wave front in the aperture acts as a source of secondary wavelets. The secondary wavelets spread out in all directions as diffracted rays in the aperture. These diffracted secondary wavelets are converged on the screen SS′ by keeping a convex lens (L) between the aperture and the screen. The screen is at the focal plane of the convex lens. Those diffracted rays traveling normal to the plane of aperture [i.e., along CP_0] are get converged at P_0.

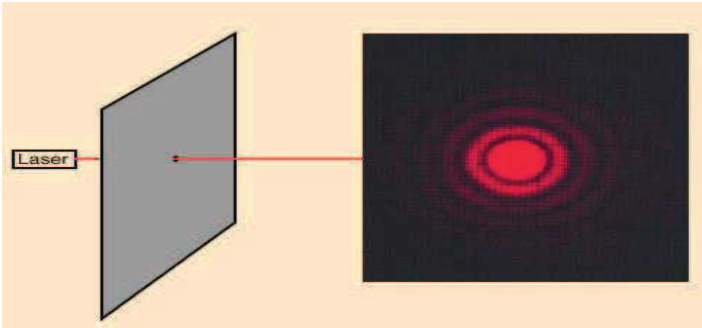

Figure 8.4

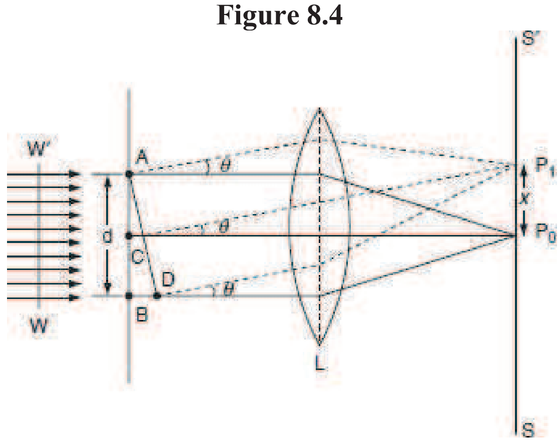

Figure 8.5

All these waves travel some distance to reach P_0 and there is no path difference between these rays. Hence a bright spot is formed at P_0 known as Airy's disc. P_0 corresponds to the central maximum.

Next consider the secondary waves traveling at an angle θ with respect to the direction of CP_0. All these secondary waves travel in the form of a cone and hence, they form a diffracted ring on the screen. The radius of that ring is x and its center is at P_0. Now consider a point P_1 on the ring, the intensity of light at P_1 depends on the path difference between the waves at A and B to reach P_1. The path difference is BD = AB sin θ = d sin θ. The diffraction due to a circular aperture is similar to the diffraction due to a single slit. Hence, the intensity at P_1 depends on the path difference d sin θ. If the path difference is an integral multiple of λ then intensity at P_1 is minimum. On the other hand, if the path difference is in odd multiples of images, then the intensity is maximum.

i.e., $d \sin \theta = n\lambda$, for minima .... (8.13)

and $d \sin \theta = (2n-1)\frac{\lambda}{2}$, for maxima .... (8.14)

Where n = 1, 2, 3… etc. n = 0 corresponds to central maximum.

The Airy disc is surrounded by alternate bright and dark concentric rings, called the Airy's rings. The intensity of the dark ring is zero and the intensity of the bright ring decreases as we go radially from P_0 on the screen. If the collecting lens (L) is very near to the circular aperture or the screen is at a large distance from the lens, then

$$Sin\,\theta \approx \theta \approx \frac{x}{f} \qquad .... \ (8.15)$$

Where, f is the focal length of the lens.

Also from the condition for first secondary minimum [using equation (8.13)]

$$Sin\,\theta \approx \theta \approx \frac{\lambda}{d} \qquad .... \ (8.16)$$

Equations (8.15) and (8.16) are equal

$$\frac{x}{f} = \frac{\lambda}{d} \ or \ x = \frac{f\lambda}{d} \qquad .... \ (8.17)$$

But according to Airy, the exact value of x is

$$x = \frac{1.22\,f\lambda}{d} \qquad .... \ (8.18)$$

Using equation (8.18) the radius of Airy's disc can be obtained. Also from this equation we know that the radius of Airy's disc is inversely proportional to the diameter of the aperture. Hence by decreasing the diameter of aperture, the size of Airy's disc increases.

8.7 DIFFRACTION DUE TO A PLANE DIFFRACTION GRATING OF N PARALLEL SLITS

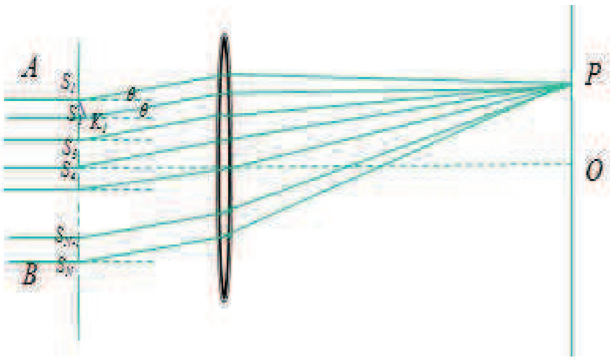

Figure 8.6

Here, S_1, S_2, S_3….. $S_N$are N narrow slits, in between points A and B. Let b= width of slit, d= width of opaque part between two slits.

The amplitude from each slit in the direction θ is

$$R_0 = \frac{A\sin\alpha}{\alpha}$$

Where $\alpha = \frac{\pi b}{\lambda}\sin\theta$ (As derived, in case of Single slit Fraunhofer diffraction)

The path difference between the wavelets from S_1 and S_2 in the direction θ is

$$S_2K_1 = (b+d)\sin\theta$$

Hence the phase difference between them

$$\frac{2\pi}{\lambda}(b+d)\sin\theta = 2\beta \text{, say}$$

If N be the total number of slits in the grating, the resultant amplitude in the direction of θ will be

$$R = R_0 \frac{\sin N\beta}{\sin\beta} = \left(\frac{A\sin\alpha}{\alpha}\right)\frac{\sin N\beta}{\sin\beta} \qquad \text{....... (8.19)}$$

Thus, the resultant intensity at point P is

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \qquad \text{....... (8.20)}$$

The factor $A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$ gives the intensity distribution due to single slit, while $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ gives the distribution of intensity in the diffraction pattern due to the interference in the waves due to N slits.

Principal Maxima

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

The intensity will be maximum when

$$\sin\beta = 0 \Rightarrow \beta = \pm n\pi$$

$Where,\ n = 0,1,2,3.....$

This result in

$$\frac{\sin N\beta}{\sin\beta} = \frac{0}{0} \text{ (Indeterminate)}$$

Applying L’ Hospital rule

$$\lim_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$\lim_{\beta\to\pm n\pi}\frac{N\cos N\beta}{\cos\beta} \Rightarrow \pm N$$

This result in

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 N^2$$

The condition for principal maxima is

$$sin\,\beta = 0 \qquad \text{or } \beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(b+d)Sin\,\theta = \pm n\pi$$

$$(b+d)\,Sin\theta = \pm n\lambda \qquad \text{.......} \quad (8.21)$$

For n = 0, we get $\theta = 0$ and this gives the direction of zero order principal maxima. The value of n = 1, 2, 3 etc. gives the direction of first, second, third etc. order principal maxima.

Minima

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

The intensity will be minimum when

$$\sin N\beta = 0 \text{ but } \sin\beta \neq 0$$

Therefore, $N\beta = \pm m\pi$ .... (8.22)

8.7.1 Missing Orders

As the resultant intensity due to N-parallel slits (plane diffraction grating) is given by

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

Where, $\alpha = \frac{\pi b}{\lambda}\sin\theta$

And $\beta = \frac{\pi}{\lambda}(b+d)\sin\theta$

Now the direction of principal maxima in grating spectrum is given as

$$(b+d)\,Sin\theta = n\lambda \qquad \text{.......} \quad (8.23)$$

Further the direction of minima of a single slit pattern is

$$b\,Sin\,\theta = m\lambda \qquad \text{.......} \quad (8.24)$$

Where *m= 1, 2, 3……*

If both the conditions are simultaneously satisfied, a particular maximum of order n will be absent in the grating spectrum, these are known as absent spectra (or missing order spectrum).

Dividing equation (8.23) by equation (8.24), we get

$$\frac{b+d}{b} = \frac{n}{m} \qquad \text{....... (8.25)}$$

If b = d, then 2nd, 4th, 6th etc. orders maxima will be missing in the grating diffraction pattern.

If d = 2b, then 3rd, 6th, 9th etc. orders maxima will be missing in the grating diffraction pattern.

8.7.2 Maximum Number of Order Available in a Grating

The grating equation is $(b+d)Sin\theta = n\lambda$

$$or \qquad n = \frac{(b+d)\,Sin\,\theta}{\lambda} \qquad \text{....... (8.26)}$$

Maximum possible value of θ is 90^0.

Therefore, Maximum possible order will be

$$n_{\max} = \frac{(b+d)Sin\ 90}{\lambda} = \frac{(b+d)}{\lambda} \qquad \text{....... (8.27)}$$

8.8 SOLVED EXAMPLES

Example 8.1: A single slit is illuminated by two wavelengths λ_1 and λ_2. One observes that due to Fraunhofer diffraction the first minimum for λ_1 coincides with the second diffraction minimum for λ_2. What is the relation between λ_1 and λ_2.

Solution: In a single slit diffraction pattern, the direction of minimum intensities are given as

$a \sin\theta = \pm m\lambda_n$, where m= 1, 2, 3 …..

Hence for m = 1, we have, $a \sin\theta = \pm\lambda_1$

and for m = 2, we have, $a \sin\theta = \pm 2\lambda_2$

Equating above two equations, we get, $\lambda_1 = \lambda_2$

Example 8.2: In a double slit Fraunhofer diffraction pattern, the screen is placed 170 cm away from the slits. The width of the slit is 0.08 mm and slits are 0.4 mm apart. Calculate of the wavelength of light, if the fringe width is 0.25 cm. Also find the missing order.

Solution: In a double slit Fraunhofer diffraction pattern, the fringe width is given by-

$$w = \frac{D\lambda}{2d}$$

Here D=170 cm=1.7 m, W= 0.25 cm= 2.5 x 10^{-3}m, a=0.08 mm = 8 x 10^{-5} m and b = 0.4 mm= 4 x 10^{-4} m, 2d = b = 4 x 10^{-4} m

$$\therefore \qquad \lambda = \frac{2dW}{D} = 0.5882 \times 10^{-6} = 5882\ \text{Å}$$

The condition for missing order is-

$$\frac{a+b}{a}=\frac{n}{m} \quad or \quad n=\left(\frac{a+b}{a}\right)m \;=\; \left(\frac{8\times10^{-5}+4\times10^{-4}}{8\times10^{-5}}\right)m = 6m$$

n=*6m*

Hence the missing orders are 6, 12, 18, 24, 30…..

8.9 SUMMARY

The basics of the diffraction phenomena along with various classes of diffraction have been discussed. The Fraunhofer diffraction for single slit, double slit, circular aperture and N slits (grating) have been discussed in the details. The calculation for the intensity of the principal maxima, secondary maxima and minima has been derived. Their relative comparison in terms of their intensities has also been made. Determination of missing orders in case of double slit and N slits (grating) diffraction pattern has also been made.

8.10 GLOSSARY

Fraunhofer Diffraction- Far field diffraction

Grating- Fine and equidistant slits in large number

Missing Order- Absent maxima

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8.13 TERMINAL QUESTIONS

Objective Type

1. Grating element is equal to

A. $n\lambda/\sin\theta$ B. $n\lambda$ C. $\sin\theta$ D. $\cos\theta$

2. In Fraunhofer's diffraction, incident light waves have________ type of wavefront.

A. Circular B. Spherical C. Cylindrical D. Plane

3. In single-slit experiment, if the red color is replaced by blue then_____________.

A. The diffraction pattern becomes narrower and crowded together

B. The diffraction bands become wider

C. The diffraction pattern does not change

D. The diffraction pattern disappears.

4. On increasing the width of a single slit, the width of the central maximum

A. increases B. remains constant C. decreases D. becomes zero

5. Maximum number of orders possible with a grating is

A. Independent of grating element

B. Inversely proportional to grating element

C. Directly proportional to grating element

D. Directly proportional to wavelength.

6. When white light is incident on a diffraction grating, the light diffracted more will be

A. Blue B. Yellow C. Violet D. Red

7. Diffraction phenomena are usually divided into __________ classes.

A. One B. Two C. Three D. Four.

8. Light of Wavelength 5000 Å is incident on a single slit of width 0.1 mm. The screen is at a distance of 2 m from the slit. The width of the central bright fringe on the screen will be

A. 18 mm B. 36 mm C. 20 mm D. 6 mm

9. Light of Wavelength 6000 Å is incident normally on a single slit of width 24 x 10^{-5} cm. The angular position of the second minimum from the central minimum from the central maximum will be -

A. 30^0 B. 60^0 C. 90^0 D. 45^0

10. In a diffraction grating, the condition for principal maxima is

A. $b \sin \theta = n\lambda$ B. $(b + d) \sin \theta = n\lambda$

C. $d \sin \theta = n\lambda$ D. $\sin \theta = n\lambda$.

Long Answer Type

1. Define diffraction phenomena. What do you mean by the Fresnel class and Fraunhofer class of diffraction?
2. Describe Fraunhofer diffraction due to single slit for central maxima and prove that the relative intensities of the successive maximum are nearly 1:1/22:1/61…

3. What are missing orders in double slit Faunhofer diffraction? Further in a grating, if the widths of transparencies and opacities are equal.
4. Give an account of the diffraction effects produced by a slit. Explain what happens when the slit width is gradually increased and also when the screen is gradually moved away from the slit.
5. Discuss Fraunhofer diffraction at a circular slit; describe the formation of Airy's disc.
6. Give the theory of a plain transmission grating. What particular spectra would be absent if the widths of transparencies and opacities of the grating are equal.

Numerical Questions

1. A circular aperture of 1.2 mm diameter is illuminated by a plane wave of monochromatic light. The diffracted light is received on a distant screen which is gradually moved towards the aperture. The center of the circular path of the light first becomes dark when the screen is 30 cm from the aperture. Calculate he wavelength of light.
2. Light of wavelength 5500 Å falls normally on a slit of width 22 × 10-5 cm. Calculate the angular position of the first two minima on either side of the central maximum.
3. Plane wave of wavelength 6×10^{-5} cm fall normally on a slit of width 0.2 mm. Calculate (i) the total angular width of the central maximum (ii) the linear width of the central maximum on a screen placed 2 m away.
4. Calculate the angle at which the first dark band and the next bright band are formed in the Fraunhofer diffraction pattern of a slit 0.3 mm wide (λ = 5890 Å).
5. In a single slit diffraction pattern the distance between the first minimum on the right and first minimum on the left is 5.2 mm. The screen on which the pattern is displayed is 80 cm from the slit and the wavelength is 5460 Å. Calculate the slit width.
6. Calculate the wavelength of light whose first diffraction maximum in the diffraction pattern due to a single slit falls at $\theta = 30^0$ and coincides with the first minimum for the red light of wavelength 6500 Å.
7. Light of wavelength 600 nm is incident normally on a diffraction grating. Two adjacent maxima occur at angles given by $\sin \theta = 0.2$ and $\sin \theta = 0.3$. The fourth-order maxima are missing. (a) What is the separation between adjacent slits? (b) What is the smallest slit width this grating can have? For that slit width, what are the (c) largest, (d) second largest, and (e) third largest values of the order number m of the maxima produced by the grating?
8. A diffraction grating is made up of slits of width 300 nm with separation 900 nm. The grating is illuminated by monochromatic plane waves of wavelength λ= 600 nm at normal incidence. How many maxima are there in the full diffraction pattern?

8.14 ANSWERS

Objective Type

1 (A), 2 (D), 3 (A), 4 (C), 5 (C), 6 (C), 7 (C), 8(C), 9(A), 10(B)

Numerical Questions

1. 6000 Å
2. 14^0 29' & 30^o
3. 6×10^{-3} radians & 1.2 cm
4. 0.112^0& 0.168^0
5. 1.68×10^{-4} cm
6. 4333.3 Å

a. 6μm (b) 1.5 μm (c) 9 (d) 7 (e) 6

7. 3

UNIT 9: RESOLUTION AND RESOLVING POWER

CONTANTS

9.1 INTRODUCTION

When the two objects are very near to each other or they are at very large distance from our eye, the eye may not be able to see them as separate. If we want to see them separate, optical instruments such as telescope, microscope etc. (for close objects) and prism and grating etc. (for spectral lines) are employed. Even if we assume that the instruments employed are completely free from all optical defects, the image of a point object or line is not simply a point or line but it is a diffraction pattern with a bright central maximum and other secondary maxima, having minima in between of rapidly decreasing intensity. Thus an optical instrument is said to be able to resolve two point objects if the corresponding diffraction patterns are distinguishable from each other.

The ability of an optical instrument to resolve (i.e. view separately) the images of two close point source is known as resolving power.

Limit of Resolution: The minimum separation between two objects that can be resolved by an optical instrument is called the limit of resolution (or just resolution).

9.2 OBJECTIVE

After studying this unit, you will be able to –

- have the basic idea of resolution.
- know the Rayleigh criterion of resolution.
- calculate the resolving power of various instruments/ accessories like grating, prism, telescope and microscope.

9.3 RAYLEIGH CRITERION OF RESOLUTION

According to Rayleigh, two close point objects are said to be just resolved if the principal maxima of one coincides with the first minima of the other and vice-versa.

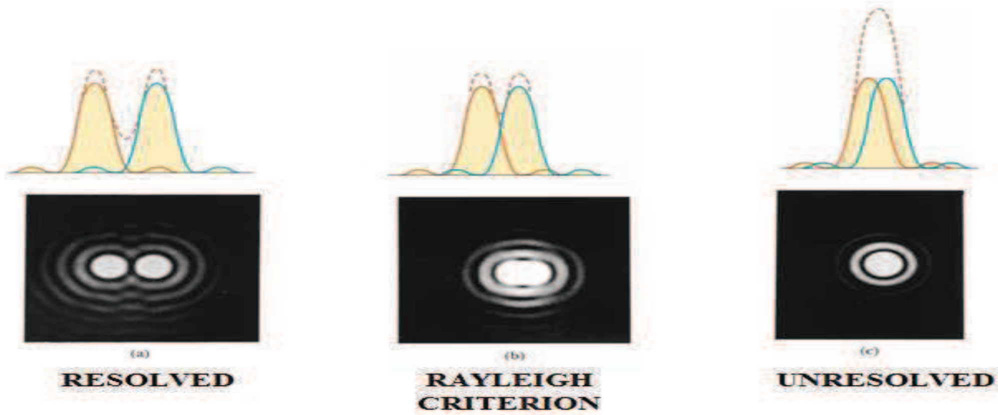

Figure 9.1

Equating the path covered by ray 1 and 2

$$DA + AE = \mu(BC) = \mu\, t \qquad (\text{for } \lambda) \qquad \ldots\ldots. \ (9.4)$$

$$DA + AF = (\mu - d\mu)\, BC = (\mu - d\mu)t \qquad (\text{for } \lambda + d\lambda) \qquad \ldots\ldots. \ (9.5)$$

Equations (9.4) and (9.5) are obtained by applying the Fermat's principle which states that for any wavelength all the actual optical paths between the incident and the emergent wavefronts must be equal. Subtracting equation (9.5) from equation (9.4), we get,

$$AE - AF = d\mu.(AC) = d\mu.t$$

From the geometry of the figure

$$AE - AF = AE - AG = d\mu.t \qquad (\text{Since } AF = AG, \text{ approximately})$$

or $GE = d\mu.t$

If GE = λ, then according to the theory of Fraunhofer diffraction, Rayleigh criterion of resolution is satisfied and spectral lines of wavelengths λ and λ + dλ, will be just resolved.

Thus $\lambda = t.d\mu$

Dividing both sides by dλ, we obtain the expression for the resolving power of prism will be

$$\frac{\lambda}{d\lambda} = t\left(\frac{d\mu}{d\lambda}\right) \qquad \ldots\ldots. \ (9.6)$$

From equation (3), it is evident that the resolving power of a prism varies directly as

(i) t, the width of the base of the prism, and
(ii) dμ/dλ, rate of change of refractive index with wavelength.

9.6 RESOLVING POWER OF TELESCOPE

A telescope is used to see the distinct objects. The details which it gives depend on the angle subtended at its objective by two point objects and not on the linear separation between them. The resolving power of a telescope is defined as the reciprocal of the smallest angle subtended at the objective by the two distinct object points which can be just seen as separate ones through the telescope.

Expression for the resolving power: Let d isthe diameter of the objective of the telescope (Fig. 4). Consider the incident rays of light from two neighboring points(say two stars lying very close to each other, not shown in the figure). Suppose dθ is the angle subtended by the two distant objects at the objective of the telescope. The ring supporting the telescope objective and the lens itself serveas a circular aperture and produce Fraunhofer diffraction patterns in the focal plane of the objective.

Let P_1 and P_2 be the positions of the central maxima of the two images. The pattern will be very close to each other with a large amount of overlapping. If the overlapping is too much, the telescope may not be able to distinguish them as separate. According to Rayleigh's criterion, the patterns will be just resolved if the central maxima of one just falls on the first minima of the other.

Now the secondary waves travelling in direction AP_2 and BP_2 meet at P_2 and have a path difference equal to $(BP_2-AP_2) = BC = d.d\theta$

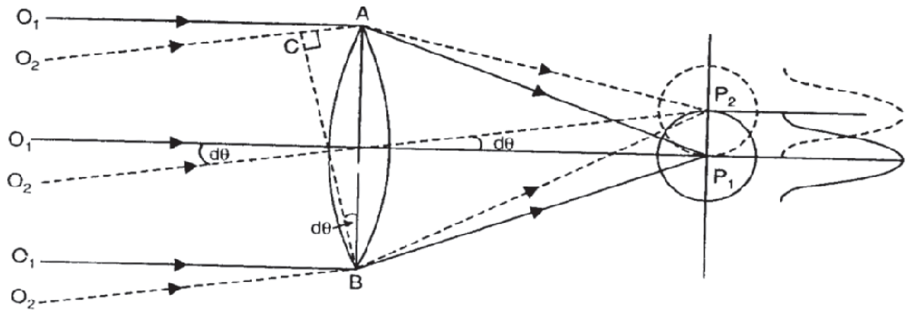

Figure 9.4

BC=AB sin dθ = AB.dθ (for small angles)

If this path difference $d.d\theta=\lambda$, the position of P_2 corresponds to the first minimum of first image and we have,

$$d.d\theta = \lambda \ \ or \ d\theta = \frac{\lambda}{d} \qquad .... (9.7)$$

The above idea may be understood in the following way:

If we consider that the whole wavefront AB isdivided into two halves AO and OB, then the path difference between the secondary waves from the corresponding points in the two halves is $\lambda/2$. All the secondary waves from the two halves interfere destructively with one another and hence P_2 corresponds to the first minimum of the first image.

The condition (9.7) holds good for rectangular aperture. According to Airy this condition in case of a circular aperture can be expressed as

$$d\theta = \frac{1.22\lambda}{d} \qquad .... (9.8)$$

Here dθ represents the minimum resolvable angle between the two distant point objects or this gives the limit of resolution of the telescope. The reciprocal of dθ measures the resolving power of the telescope. Hence

$$\frac{1}{d\theta} = \frac{d}{1.22\lambda} \qquad .... (9.9)$$

Thus a telescope with large diameter of objectivehas a higher resolving power.

9.7 RESOLVING POWER OF MICROSCOPE

The function of a microscope is to magnify an object and give its finer details which cannot be observed by naked eye. The ability of a microscope to form distinctly separate images of two neighboring small objects is known as its resolving power. It is measured by the smallest linear separation between two point objects whose images are just resolved by the objective of the microscope. The smaller is the linear separation which can be resolved, the higher is said to be the resolving power.

Expression for Resolving Power: In Figure 9.5, AB is the aperture of the objective of the microscope; O_1 and O_2 are the self-luminous point objects very close to each other and separated at a distance d. The periphery of the objective acts as a circular aperture and as a result the images of O_1 and O_2 are Fraunhofer diffraction patterns. The patterns consisting of a central bright disc surrounded by a series of alternate dark and bright rings.P_1 represents the central maximum of the diffraction pattern of the point object O_1. Similarly P_2 represents the central maximum of the diffraction pattern of the other point object O_2.

According to the Rayleigh's criterion the two objects may be resolved if the central maximum of one pattern falls on the first minimum of the other. In this case the two objects may be resolved if P_1 islocated at the first minima of the diffraction pattern centered at P_2.

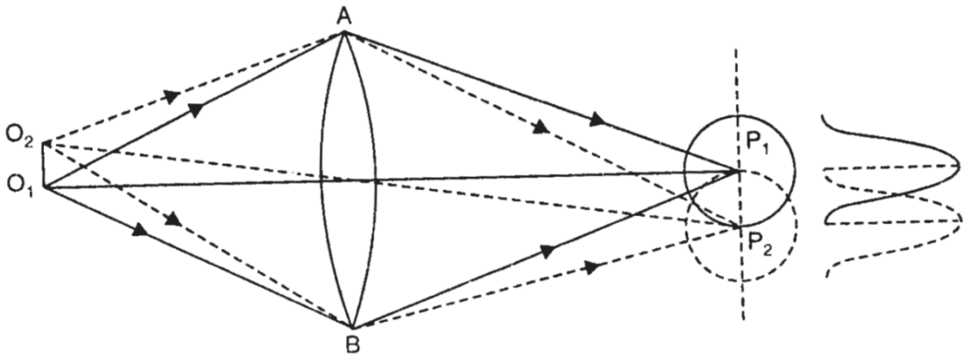

Figure 9.5

According to the Rayleigh's criterion the two objects may be resolved if the central maximum of one pattern falls on the first minimum of the other. In this case the two objects may be resolved if P_1 islocated at the first minima of the diffraction pattern centered at P_2. Thus we have to find out condition under which the first minima of the diffraction pattern due to O_2 lies at the central maxima of diffraction pattern due to O_1. This will happen when the path difference between the extreme rays O_2BP_1 and O_2AP_2 is equal to λ. To consider this path difference, the magnified view of O_1O_2 and the rays starting from them are shown in Fig. 6. The path difference is given by

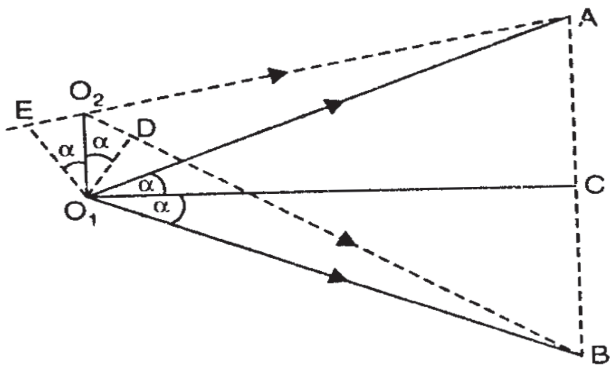

Figure 9.6

(O_2B+BP_1)- $(O_2A + AP_1) = O_2B$- O_2A (Since $BP_1=AP_1$)

In Figure 9.6,O_1C is perpendicular to CA and O_1D is perpendicular to O_2B.

O_2B- $O_2A = (O_2D + DB)$- $(EA$-$EO_2) =O_2D + EO_2$(As $DB= O_1B= O_1A=EA$)

Therefore, path difference = $O_2D + EO_2 = 2d \sin \alpha$

If the path difference 2d sin α = 1.22λ, then P_1 corresponds to the first minimum of the image P_2 and the two images appear just resolved.

$$2d \sin \alpha = 1.22\lambda$$

or

$$d = 1.22\lambda/ 2\sin \alpha \qquad \text{....... (9.10)}$$

The result is derived on the assumptions that the objects viewed with microscope are self-luminous and emitting light of wavelength λ.

In case of objects illuminated by some external source of light of wavelength λ, Abbe showed that the factor 1.22 may be omitted and we can write

$$d = \lambda/ 2\sin \alpha \qquad \text{....... (9.11)}$$

The high resolution power microscopes are generally oil immersion types in which the space between the object and objective is filled with an oil of refractive index μ. In this case as the path difference will then be multiplied with the factor μ

$$d = \lambda/ 2\mu\sin \alpha \qquad \text{....... (9.12)}$$

Here, the factor μsin α is known as the numerical aperture of the microscope.

$$\therefore \text{Resolving power of the microscope} = \frac{1}{d} = \frac{2\mu \sin \alpha}{1.22\,\lambda} \qquad \text{....... (9.13)}$$

Thus, using small wavelengths (UV) and using quartz lenses, the resolving power of the microscope can be increased. Such microscopes are known as the ultra-microscope.

9.8 SOLVED EXAMPLES

Example 9.1: Calculate the minimum number of lines in a grating which will just resolve the sodium lines in the first order spectrum. The wavelengths are 5890 and 5896 Å.

Solution: We know that resolving power λ/dλ = nN

Here n=1, λ_1 = 5890 Å = 5890×10^{-8} cm

∴ $d\lambda = 5896 \times 10^{-8} - 5890 \times 10^{-8} = 6 \times 10^{-8}$ cm.

Now, N=(1/n) x (λ/dλ) = $(5890 \times 10^{-8})/(1 \times 6 \times 10^{-8})$ =982 approximately

Example 9.2: A grating has 15 cm of the surface ruled with 16000 lines per cm. What is the resolving power of the grating in the first order?

Solution: The resolving power of a grating is given by-

λ/dλ = nN, here n = 1, N = 15 x 6000 = 90000

λ/dλ = 1 x 90000 =90000

Example 9.3: A prism spectrometer uses a prism of base 5 cm and material whose dispersion $\frac{d\mu}{d\lambda}$ is 200 in the range λ = 5000Å. What is the smallest difference of the wavelength in this range which this spectrometer may resolve?

Solution: The expression for the resolving power of prism is

$$\frac{\lambda}{d\lambda} = t\left(\frac{\mathrm{d}\mu}{\mathrm{d}\lambda}\right),$$

Here, t= 5 cm, λ = 5000Å = 5 x 0^{-5} cm, $\frac{d\mu}{d\lambda} = 200$

Putting the values, we get, dλ =5 x 10^{-8} cm =5 Å

Example 9.4: Two pin holes 1.5 mm apart are placed in front of a source of light of wavelength 5.5 x 10^{-5} cm and seen through a telescope with objective diameter of 0.4 cm. Find the minimum distance from the telescope at which the pin holes can be resolved.

Solution: We know that $d\theta = \frac{1.22\lambda}{d}$ and also $d\theta = \frac{x}{a}$

∴ $\frac{1.22\lambda}{d} = \frac{x}{a}$ or $a = \frac{xd}{1.22\lambda}$ $= \frac{0.15 \times 0.4}{1.22 \times 5.5 \times 10^{-5}}$ = 894.2 cm

Example 9.5: The smallest object detail that can be resolved with a certail microscope with light of wavelength 6000 Å is 3.5 x 10^{-5} cm. Find (i) The numerical aperture of the objective when used dry, and (ii) The numerical aperture obtained if an immersion oil of refractive index 1.5 is used.

Solution: The resolving power of microscope is –

$$d = \frac{\lambda}{2Sin\alpha} = \frac{\lambda}{2\,NA},$$ where NA is numerical aperture

(i) NA = $NA = \frac{\lambda}{2d} = \frac{6000 \times 10^{-8}}{2 \times 3.5 \times 10^{-5}} = 0.86\ approx.$

(ii) Oil immersion numerical aperture= μ x dry aperture = 1.5 x 0.86 =1.44

9.9 SUMMARY

The resolving power of an optical instrument is defined as its ability to just resolve the images of two close point sources or small objects. The Rayleigh Criterion gives a quantitative account of the phenomena of resolution. The definitions and physical meanings for the resolving powers of diffraction grating, prism, telescope and microscope were discussed. Their mathematical expressions have also been derived in the present chapter.

9.10 GLOSSARY

Resolving Power: Ability of an optical instrument to see the close objects separately

Limit of resolution: Minimum resolvable distance

Principal maxima: Central maxima

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9.12 SUGGESTED READINGS

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9.13 TERMINAL QUESTIONS

Objective Type

1.The maximum resolving power of a microscope can be obtained with

(A) Violet light (B) Yellow Light (C) Red Light (D) Green Light

2.What will be limit of resolution of a microscope if its numerical aperture is 0.5 and the wavelength of light used 5000 Å

(A) 6100 mm (B) 6100 cm (C) 6100 m (D) 6100 Å

3.Two stars distant eight light years are just resolved by a telescope. The diameter of the telescope lens is 26 cm. If the wavelength of the light used is 5000 Å , the minimum distance between the stars will be

(A) 1.95×10^{12} M (B) 1.95×10^{11} M (C) 1.95×10^{10} M (D) 1.95×10^{9} M

4.The resolving power of a telescope can be increased by having a

(A) Large focal length of eyepiece (B) Small focal length of eyepiece (C)Large aperture of objective lens (D) Small aperture of objective lens

5.The Resolving power of a grating having N slits in nth order will be

(A) (n+N) (B) (n-N) (C) nN (D) n/N

6.The resolving power of a prism is

(A) Directly proportional to the rate of change of refractive index with wavelength
(B) Inversely proportional to rate of change of refractive index with wavelength
(C) Inversely proportional to the thickness of the prism
(D) Independent of thickness of prism

Long Answer Type

1. Discuss Rayleigh criterion for resolution. What is limit of resolution? Determine an expression for the resolving power of a grating.
2. Explain clearly, what is meant by the resolving power of an optical instrument and deduce an expression for the resolving power of a prism.
3. Explain what do you understand by the limit of resolution of a telescope and obtain an expression for it. What is the effect of the size of the image of a star as aperture of the objective increases?
4. On the basis of diffraction theory, explain the need of large apertures for telescopes used for astronomical purposes.
5. Define the resolving power of a microscope. Deduce an expression for it and discuss it.

Numerical Questions

1. Find the separation of the two points that can be resolved by a 500 cm telescope. The distance of the moon is 3.8 x 10^5 KM. The eye is highly sensitive to light of wavelength of 5500 Å.
2. Show that for a transmission grating with 1 inch ruled space, the resolving power cannot exceed 5 x 10^4 at normal dence for λ = 5080 Å.
3. A microscope objective gathers light over a cone of semi-angle 30^0 and uses visible light of 5500 Å. Estimate its resolving limit.
4. Calculate the minimum thick ness of the base of a prism which will just resolve the D1 and D2 lines of sodium. Given μ for wavelength 6563 Å = 1.6545 and for wavelength 5270 Å = 1.6635.
5. Calculate the resolving power of a prism which has a dispersion $\frac{d\mu}{d\lambda}$ = 600 per cm and a base of 3 cm. Will this be adequate to resolve two spectral lines (i) 5890 Å (ii) 5230 Å
6. A diffraction grating with a width of 2.0 cm contains 1000 lines/cm across that width. For an incident wavelength of 600 nm, what is the smallest wavelength difference this grating can resolve in the second order?
7. How many rulings must a 4.00-cm-wide diffraction grating have to resolve the wavelengths 415.496 and 415.487 nm in the second order? (b) At what angle are the second-order maxima found?

9.14 ANSWERS

Objective Type

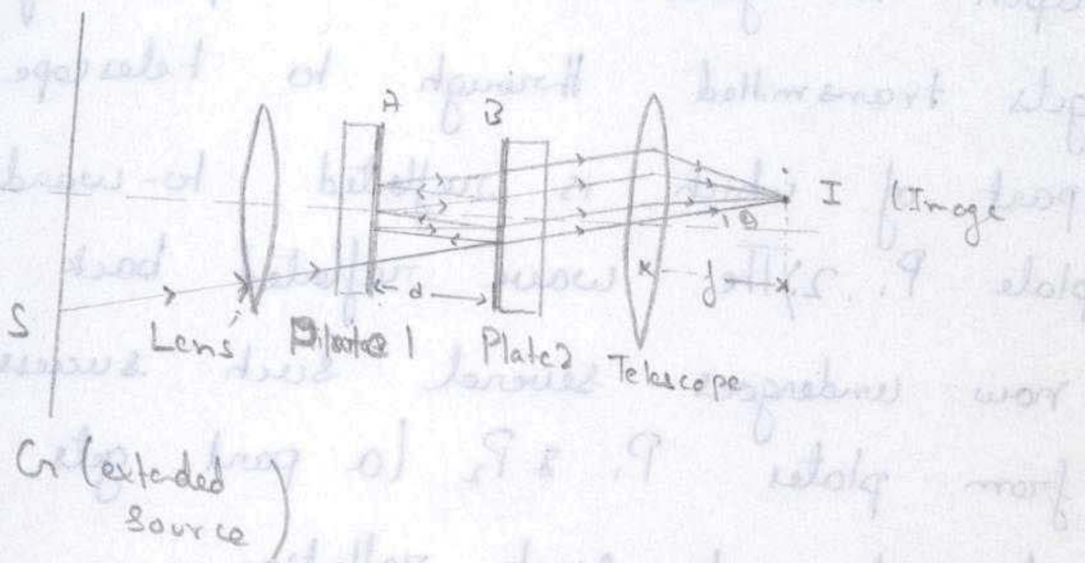
1.(A), **2**. (D), **3**. (B), **4**. (C), **5**. (C), **6**. (A)

Numerical Questions

1.61 m, **3.** 6.1×10^{-5} cm, **4.** 1.41,**5.(i)** Prism will resolve this line **(ii)** Prism will not resolve this line, **6.**$\Delta\lambda$= 0.15 nm, **7.** (a) 23100, (b) 28.7°

Q. Describe the working of a Fabry Perot interferometer. Determine the intensity of the fringes of the transmitted light. Why the fringes obtained in the Fabry Perot interferometer are comparitively sharper than those obtained from Michelson interferometer.

Ans.



Fabry Parot Interferometer

L → Convex lens in whose focal plane extended source G is kept

P_1, P_2 → Two optically plane glass plates with inner surfaces silvered & placed exactly parallel to each other. The outer surfaces are slightly prismatic. ~~to avoid~~

T → Telescope lens with focal length 'f'.

I → fringe pattern produced by source S on focal plane of lens (telescope).

Working :

1. The incident light from point source S on G is ~~made~~ to fall ~~normally upon~~ parallel upon glass plate

1. A light ray from source S on G falls upon the glass slab P_2. A part of which gets transmitted through to telescope & a part of which is reflected to-wards glass plate P_1. 2) The wave reflected back to P_1 now undergoes several such successive reflections from plates P_1 & P_2 (a part gets transmitted also at each such reflection.

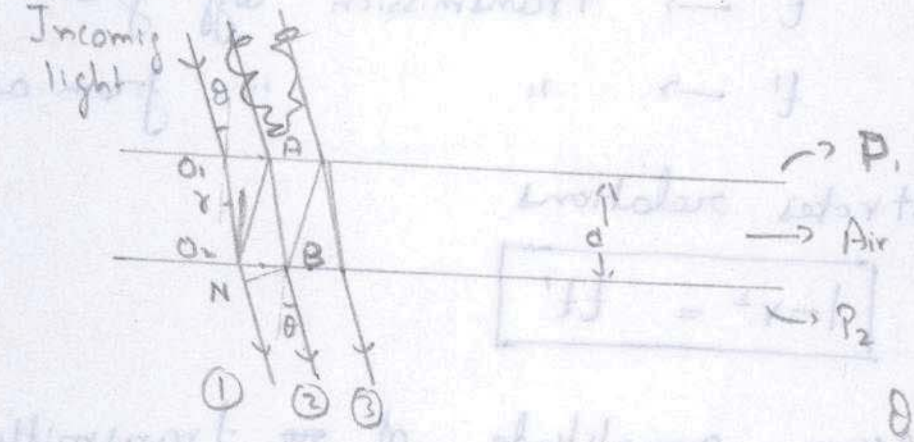
3) The transmitted waves from plate P_2 which travel towards lens T of telescope are parallel to each other & are brought to focus on screen S_2.

4) Since all these rays are from a same point source S, thus they satisfy the coherency condition and produce interference fringes at point I.

5) Thus fabry perot interferometer works on interference by "<u>division of amplitude</u>" and uses "<u>multiple beam interferometery</u>".

Fringes:

The fringes are obtained due to an air slab formed between the plates P_1 & P_2 which can be analyzed as:



θ → angle of incidence
r → angle of refraction

Every ray in transmitted light has a (constant) path difference of with successive ray given by:

$$\Delta x = O_2A + AB - O_2N$$
$$= 2\, O_2A - O_2N$$
$$= 2 \times d \sec r - 2d \tan r \sin\theta$$

Using $\frac{\sin\theta}{\sin r} = {}_1\mu_2 = 1 \Rightarrow \theta = r$.

$$\Delta x = 2d\cos\theta$$

Hence each ray towards telescope has a phase difference δ with it's successor:

$$\boxed{\delta = \frac{2\pi}{\lambda} \times \Delta x}$$

If reflection coff. of surface is r & t is transmission coff. then amplitudes of ray 1, 2, 3 etc are :

(a_1) (a_2) (a_3)

att', $att'r^2$, $att'r^4$ - - - - [a → amplitude of incoming ray]

where t → transmission coff from ~~slab~~ plate to air

t' → " " from air to slab

Using Strokes relations

$$\boxed{1-r^2 = tt'}$$

Hence sum amplitude of ~~re~~ transmitted light is

$$A = a_1 + a_2 e^{i\delta} + a_3 e^{i2\delta} + - -$$

{as each ray has phase difference of δ}

$$A = att'\left[1 + r^2 e^{-i\delta} + r^4 e^{-i2\delta} + - - \right]$$

$$A = \frac{a\,tt'}{1 - r^2 e^{-i\delta}} \quad \{ \text{as } |r^2 e^{i\delta}| < 1 \}$$

$$\therefore I = A^* A = \frac{a^2 (tt')^2}{(1 - r^2 e^{-i\delta})(1 - r^2 e^{+i\delta})} = \frac{a^2 (1-r^2)^2}{(1 + r^4 - 2r^2 \cos\delta)}$$

$$\boxed{I = \frac{a^2 (1-r^2)^2}{(1-r^2)^2 + 4r^2 \sin^2\frac{\delta}{2}} = \frac{I_0}{1 + F \sin^2\frac{\delta}{2}}}$$

where $I_0 = a^2$ (incoming intensity) $F = \left(\frac{2r}{1-r^2}\right)^2$ {Finesse Coefficient}.

Sharpness in comparision to Michelson Interferometer

From formula for I, we see that $I = I_0$ ie. maxima for $\boxed{\delta = 2m\pi}$

- Let us calculate Full Width at Half Maxima ie width within which intensity falls to half of maximum.

Let with on either sides be $\Delta\delta$

$$\therefore \frac{I_0}{2} = \frac{I_0}{1 + F\sin^2\frac{(\delta+\Delta\delta)}{2}} = \frac{I_0}{1 + F\sin^2\frac{\Delta\delta}{2}}$$

$$\left(\text{as } \frac{\delta}{2} = m\pi\right)$$

$$\therefore F\sin^2\frac{\Delta\delta}{2} = 1$$

$$\sin\frac{\Delta\delta}{2} = \frac{1}{\sqrt{F}}$$

$$\Delta\delta = \frac{2}{\sqrt{F}} \quad \text{for small } \Delta\delta$$

$$\Rightarrow FWHM = 2\Delta\delta = \frac{4}{\sqrt{F}} = \frac{4(1-r^2)}{2r}$$

if we take $r^2 = 0.8 = R$ (usual case).

$$2\Delta\delta \simeq 2\cdot2 \times 0.11 = 0.44 \text{ radians}$$

For Michelson:

$$\boxed{I = 4I_0\cos^2\frac{\delta'}{2}}$$; maxima occours for $\delta' = 2m\pi$

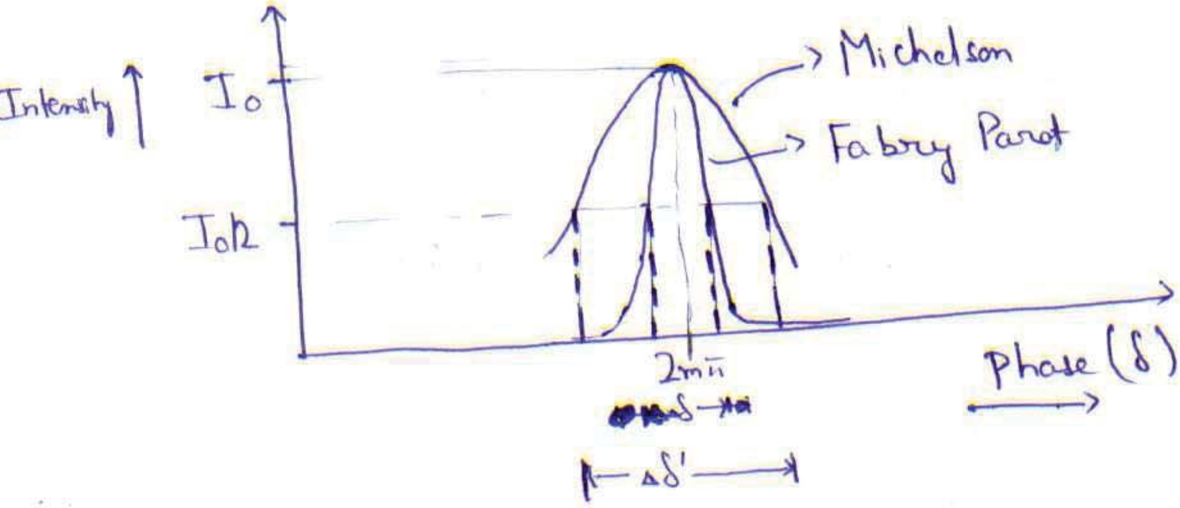
$\Rightarrow$ FWHM occours for $\cos^2\frac{\delta'}{2} = \frac{1}{2}$

$$\Rightarrow \cos^2\frac{2m\pi + \Delta\delta'}{2} = \cos^2\frac{\Delta\delta'}{2} = \frac{1}{2} \; ; \; \Delta\delta = 2\times\frac{\pi}{4}$$

$\therefore$ FWHM $= 2\Delta\delta' = 2\times 2\times \frac{\pi}{4} = \pi = 3.14$ radians.

Hence we see that Fabray Parot fringes are <u>sharper by around</u>:

$\frac{3.14}{2\times \cdot 22} \approx 7$ times than Michelson.



Hence <u>interference fringes by various multiplets of λ separated by Δλ could be viewed more clearly by fabry parot</u> interferometer due to sharpness of fringes.