Lecture Notes.

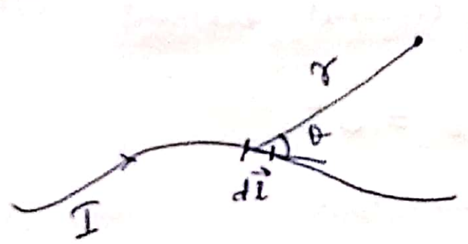
B. Sc. 2nd Sem (CBCS)

Magnetism

Biot - Savart law.

In order to find the magnetic field at any arbitrary point due to a current carrying conductor of any arbitrary shape we can use the Biot Savart law.

let us elaborate it with some diagram -



This is a current carrying conductor of any shape, conducting current I. We want to find magnetic field at P (any arbitrary point) due to current I. First we will find B (magnetic field) for small line element $\vec{dl}$.

According to Biot Savart law.
Magnetic field at P varies as follows

i) $\propto I$ (current)

ii) $\propto \frac{1}{r^2}$ ($r \rightarrow$ distance between dl and P)

iii) $\propto \sin\theta$ ($\theta \rightarrow$ Angle between $\vec{dl}$ and $\vec{r}$)

iv) $\propto dl$ (line element).

So,

$$dB \propto \frac{I\,dl \sin\theta}{r^2}$$

$$\Rightarrow \quad dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

where $\frac{\mu_0}{4\pi}$ = constant.

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\theta}{r^2}$$

⇓

Magnetic field at P due to whole length -wire.

$I\,dl$ is called current element.

case-(I) when $\theta = 0°$

$B = 0$

(II) And $\theta = 90°$

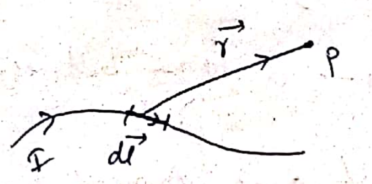
B = maximum.

<u>Direction</u>

vector form of this law.

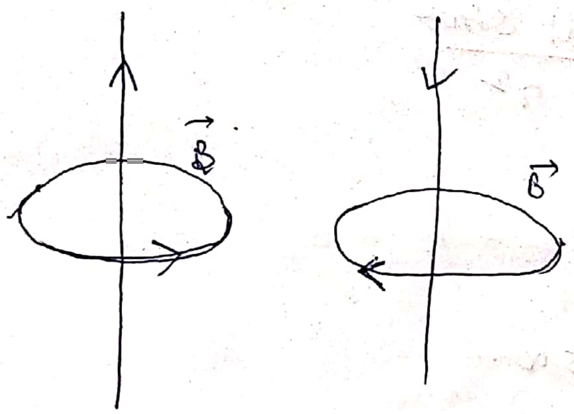
$$\vec{dB} = \frac{\mu_0 I}{4\pi} \frac{\vec{dl} \times \vec{r}}{r^2}$$

B is $\perp$ to the plane containing $\vec{r}$ and $\vec{dl}$



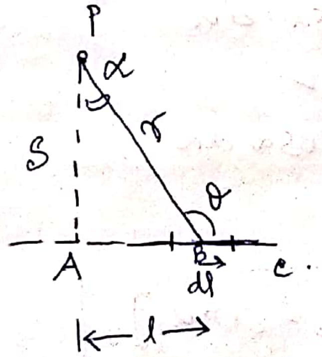
In this case direction of B is ~~up~~ $\perp$ ~~to~~ upward to the page.

If current change the direction B will also change the direction (~~Inwar~~ Into the page).



Application of Biot-Savart law.

1) Magnetic field due to straight current carrying conductor.



$\vec{dl}$ is the small element of the conductor.

First we will find dB at P due to the current element Idl. Then Integrate to find the B for whole wire.

θ → Angle between $\vec{r}$ and $\vec{dl}$

S → perpendicular distance between P and the wire.

α → Angle between $\vec{S}$ and $\vec{r}$.

According to the Biot-Savart law, dB at P.

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin\theta}{r^2}$$

From the figure.

$$\theta = (90^\circ + \alpha).$$

And, $s = r\cos\alpha$.

$$\frac{l}{s} = \tan\alpha. \qquad [AB = l]$$

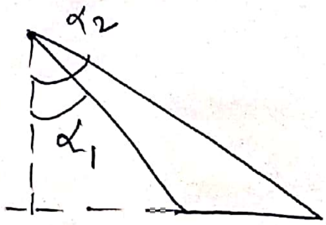
$$\Rightarrow \quad dl = s \sec^2\alpha \, d\alpha.$$

Re writting dB in terms of α and s.

$$dB = \frac{\mu_0 I}{4\pi} \frac{s \sec^2\alpha \, d\alpha \, \cos\alpha}{s^2 \sec^2\alpha}$$

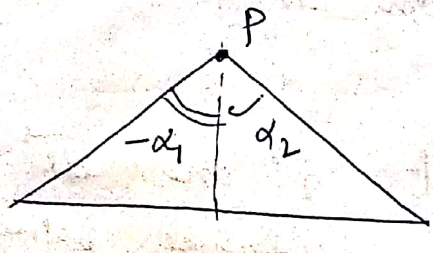
$$dB = \frac{\mu_0 I}{4\pi s} \cos\alpha \, d\alpha.$$

$$B = \frac{\mu_0 I}{4\pi s} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha.$$



$$\boxed{B = \frac{\mu_0 I}{4\pi s} [\sin\alpha_2 - \sin\alpha_1]}$$

i) For long wire,



$$B = \frac{\mu_0 I}{4\pi s} [\sin\alpha_2 + \sin\alpha_1]$$

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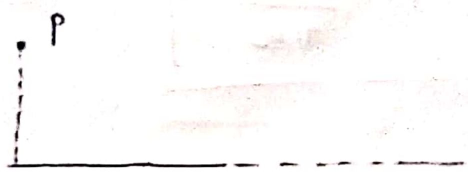
ii) For infinitely long wire

$\alpha_1 = \alpha_2 = 90^\circ$

$B = \frac{\mu_0 I}{4\pi s}(\sin 90^\circ + \sin 90^\circ)$

$$\boxed{B = \frac{\mu_0 I}{2\pi s}}$$

iii) At one end of the infinitely long wire



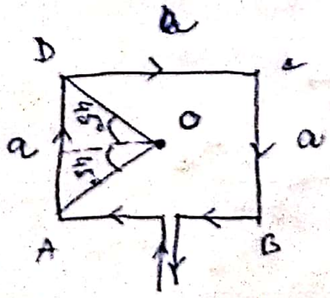
$\alpha_1 = 0^\circ, \quad \alpha_2 \approx 90^\circ$

$\therefore \quad B = \frac{\mu_0 I}{4\pi s}[0 + 1]$

$= \frac{\mu_0 I}{4\pi s}$

Problem

i) Find the magnetic field at the centre of the square shaped current carrying conductor.



Solution

B at O due to element AD.

$B_1 = \frac{\mu_0 I}{\left(\frac{a}{2}\right)4\pi}\left[\sin 45^\circ + \sin 45^\circ\right]$

$$B_1 = \frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{\mu_0 I}{2\pi a} \cdot \frac{2}{\sqrt{2}}$$

$$= \frac{\mu_0 I}{\sqrt{2}\,\pi a}$$

Total $B = B_1 + B_2 + B_3 + B_4$

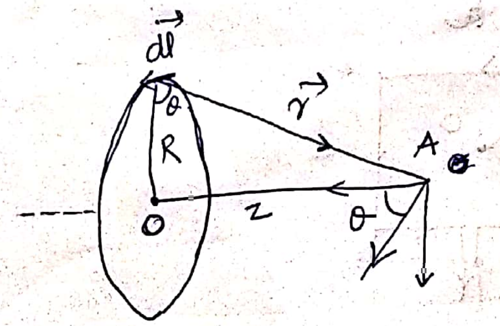
$$= 4 \times \frac{\mu_0 I}{\sqrt{2}\,\pi a}$$

$$\boxed{B = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}}$$

problem

1) Find the magnetic field at the centre of the n sided polygon, carrying a steady current I and R be the distance from the centre to any side.

2) Magnetic field on the axis of the current carrying circular coil.



First we will find $\vec{B}$ at A due to current element $I\,\vec{dl}$.

It is obvious, angle between $\vec{dl}$ and $\vec{r}$ is $90°$.
According to the Biot-Savart law.
Magnetic fied at A.

$$dB = \frac{\mu_0 I}{4\pi r^2} dl \sin 90^0$$

$$= \frac{\mu_0 I \, dl}{4\pi r^2}$$

componant along the axis $dB\cos\theta$
$\perp$ to the axis $dB\sin\theta$.

componant $\perp$ to the axis concels each other.
only $dB\cos\theta$ comp. exist.

$$\therefore \quad dB = \frac{\mu_0 I}{4\pi r^2} dl \cos\theta.$$

from the figure.

$$\cos\theta = \frac{R}{r} \Rightarrow r = \frac{R}{\cos\theta}.$$

And $\cos\theta = \frac{R}{(R^2+z^2)^{1/2}}$

$$\therefore \quad dB = \frac{\mu_0 I \, dl}{4\pi} \cdot \frac{\cos^3\theta}{R^2}$$

$$= \frac{\mu_0 I}{4\pi R^2} \cdot \frac{R^3}{(R^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 I R}{4\pi (R^2+z^2)^{3/2}} dl.$$

$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{I R}{(R^2+z^2)^{3/2}} \int dl$$

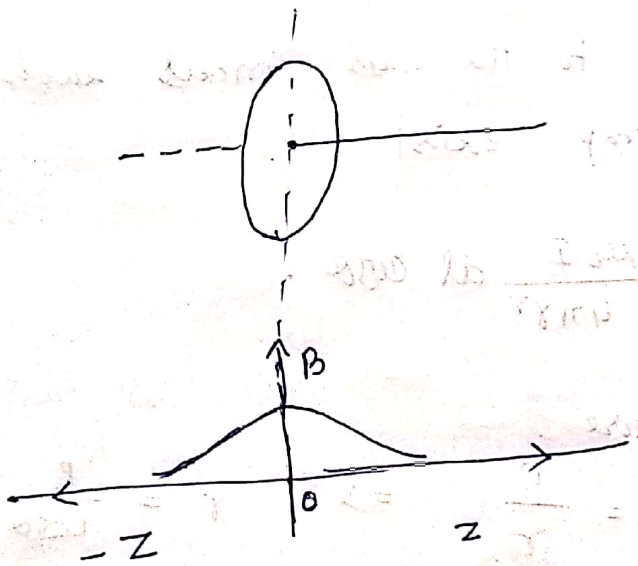
$$= \frac{\mu_0 I}{4\pi} \frac{R}{(R^2+z^2)^{3/2}} 2\pi R$$

$$\boxed{B = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}}$$

Magnetic field at the centre : $z = 0$.

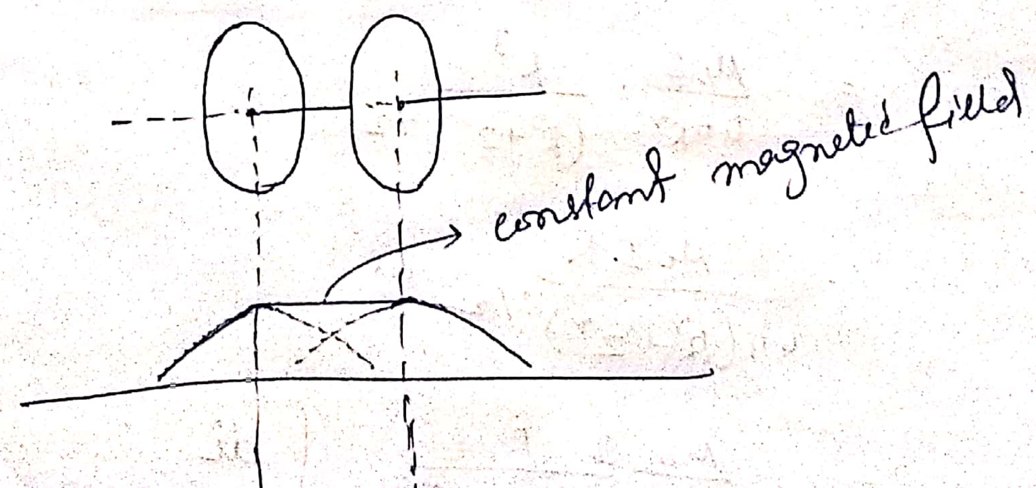
$$B = \frac{\mu_0 I R^2}{2R^3}$$

$$\boxed{B = \frac{\mu_0 I}{2R}}$$



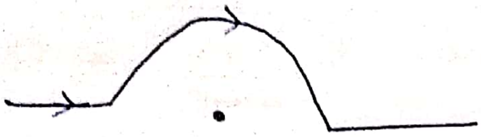
B maximum at the centre.

If there is 2 co-axial coil at the vicinity



case: →

1) Magnetic field due to half circular coil.



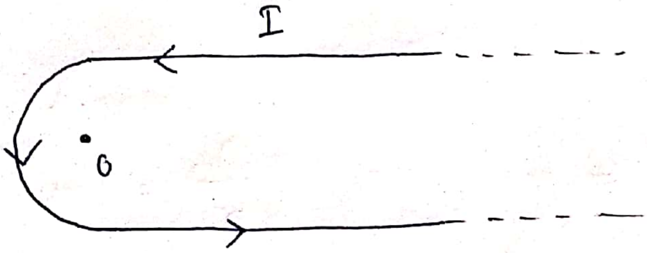
$$B = \frac{\mu_0 I}{4\pi R^2}\int dl.$$

$$= \frac{\mu_0 I}{4\pi R^2} \cdot \pi R$$

$$= \frac{\mu_0 I}{4R}.$$

problem

2)



Find the field at point o.