

Magnetic force on a moving charged particle -

of a charged particle (whether +ve or -ve) moves in a magnetic field, it experiences a magnetic force.

This force depends on the

- 1) charge of the particle
- 2) velocity of the "
- 3) strength of the magnetic field
- 4) and the direction of the motion of the particle and magnetic field.

of a q charged particle moving with velocity v in a magnetic field B , then force experienced by that particle is

$$\vec{F} = q(\vec{v} \times \vec{B}) = qvB \sin \theta \hat{n}$$

This force is called Lorentz force.

eg,

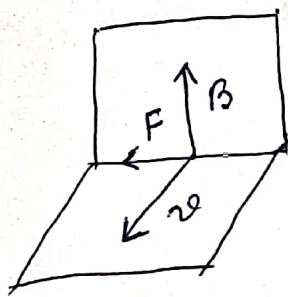
i) $q = 0$, then $F = 0$

ii) $v = 0$ " $F = 0$

iii) and $\theta = 0^\circ$ then $F = 0$.

Direction of the force

Force acting on the particle is $\perp$ to the $\vec{v}$ and $\vec{B}$ vector.



If we change the direction of v and B direction of F will be changed accordingly.

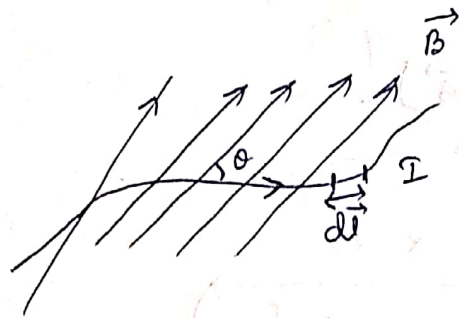
Lorentz force does not do any work.

As Lorentz force acting on the particle is $\perp$ to the ~~motion~~ direction of motion of the particle, that's why Lorentz force does not do any work. As we know,

$$\begin{aligned}
 \text{Work done} &= \int_{x_1}^{x_2} \vec{F} \cdot d\vec{x} \\
 &= \int_{x_1}^{x_2} q(\vec{v} \times \vec{B}) \cdot \frac{d\vec{x}}{dt} \cdot dt \\
 &= q \int_{x_1}^{x_2} (\vec{v} \times \vec{B}) \cdot \vec{v} \, dt \\
 &= q \int (\vec{v} \times \vec{v}) \cdot \vec{B} \, dt \\
 &= 0 \\
 &= \underline{\underline{0}}
 \end{aligned}$$

Note $\rightarrow$ This force cannot increase the k.E. of the particle, it only changes the direction of the particle motion.

Magnetic force on a current carrying wire.



We need to find force on the wire due to the presence of magnetic field $\vec{B}$.

First we will find force on $d\vec{l}$ element.
we have formula.

$$F = q(\vec{v} \times \vec{B})$$

We need to find first q and v with respect to I .

Let say line charge density is λ .

total charge in $d\vec{l}$ element is $dq = \lambda dl$

current = $\frac{\text{total charge passing through cross-section}}{\text{time (second)}}$

if $\text{time} = 1 \text{ sec}$.

current = $\frac{\text{total charge passing through a cross-section}}{\text{area}}$

= total charge in the element of length v

$$I = \lambda v$$

$$\therefore dF = dq(\vec{v} \times \vec{B})$$

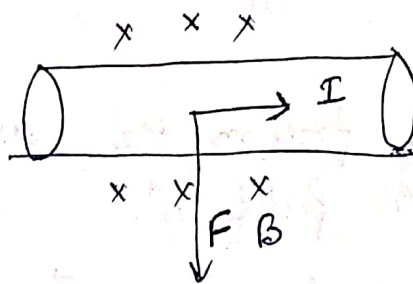
$$\Rightarrow F = \int dq(\vec{v} \times \vec{B})$$

$$= \int (\vec{v} \times \vec{B}) \lambda dl$$

$$= \int (\vec{I} \times \vec{B}) dl$$

As, I and direction of dl is same.

$$\vec{F} = \int I (d\vec{l} \times \vec{B})$$

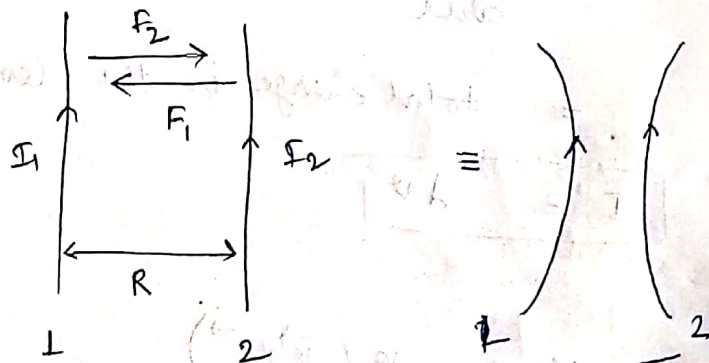


$\Rightarrow$ If length of the wire is l and the wire is straight then,

$$\vec{F} = \int I (d\vec{l} \times \vec{B})$$

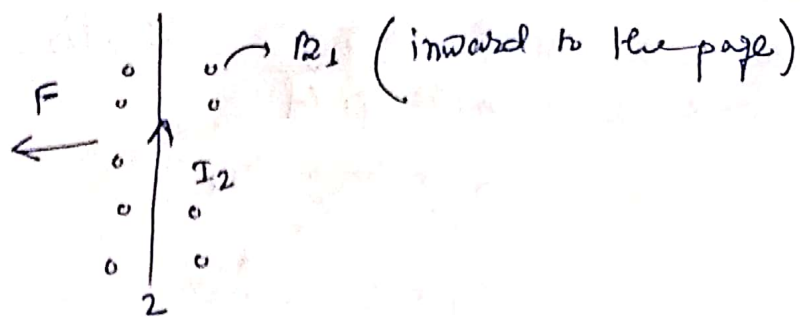
$$\vec{F} = I (\vec{l} \times \vec{B})$$

Force between two current carrying straight wire



Magnetic field B_1 due to current I_1 of wire 1 at the position of wire 2.

$$B_1 = \frac{\mu_0 I_1}{2\pi R}$$



Wire 2 carrying current I_2 immersed in a magnetic field B_1 (due to wire 1).

Force of this wire 2 is.

$$\begin{aligned}\vec{F} &= I_2 \int d\vec{l} \times \vec{B}_1 \\ &= I_2 \int dl B_1 \sin 90^\circ \hat{n} \\ &= I_2 B_1 \int dl \hat{n} \\ \vec{F} &= \frac{\mu_0 I_1 I_2}{2\pi R} \int dl \hat{n}\end{aligned}$$

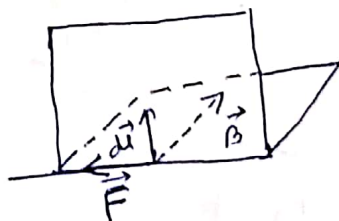
$\int dl$ = length of the wire.

$$\therefore \vec{f}_2 = \frac{\mu_0 I_1 I_2}{2\pi R} \hat{n} = \text{force per unit length.}$$

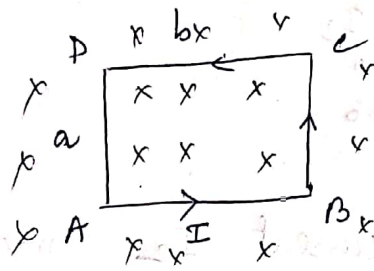
Similarly force on wire 1 due to wire 2.

$$\vec{f}_1 = \frac{\mu_0 I_1 I_2}{2\pi R} (-\hat{n})$$

$\vec{f}_1$ and $\vec{f}_2$ directed towards each other.



* Force on a current carrying rectangular loop in the presence of magnetic field



Let say a rectangular loop of ~~side~~ side a and b .
 Direction of the plane of the loop is $\hat{n}$, which is $\perp$ to the plane of the loop.

If a magnetic field B is applied along the direction of $\hat{n}$, means the $\perp$ of the plane of the loop, we need to find the ~~the~~ total force acting on the loop due to this B .

Force of AB element.

$$\vec{F}_1 = I \int d\vec{l} \times \vec{B}$$

$$|F_1| = IB \sin \theta \int dl$$

$$= IBb \quad [\text{as } \theta = 90^\circ]$$

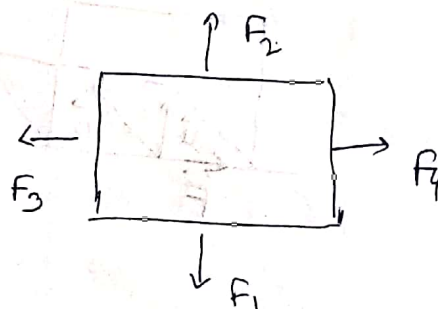
Similarly force on CD element

$$|F_2| = IBb = |F_1|$$

Force on DA element $|F_3| = IBa$

and BC element $|F_4| = IBa = |F_3|$

Direction of these forces as shown in the figure



F_1 and F_2 counter ~~act~~ Balance to each other
 $\vec{F}_1 = -\vec{F}_2$

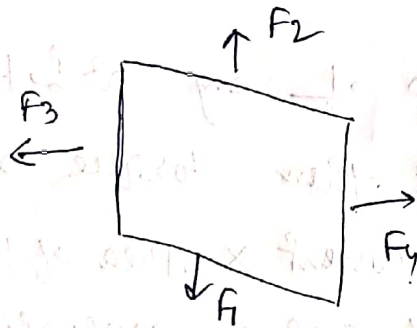
and $\vec{F}_3 = -\vec{F}_4$

Net force on that loop is zero.

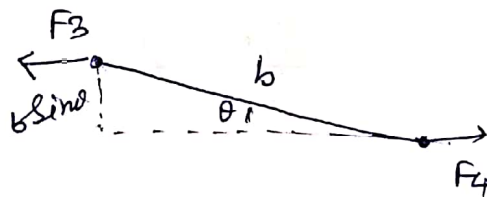
Therefore there will be no movement of this loop due to the presence of magnetic field.

* What will happen if we slightly tilt the loop such that AD and BC remains $\perp$ to $\vec{B}$ but ~~not~~ CD and AB not. i.e. $\vec{n}$ and $\vec{B}$ makes an angle θ . [not 0 as earlier case]

Force on AD element $F_3 = I a B$
 " " BC " $F_4 = I a B$



F_3 and F_4 equal and opposite but do not share same line of action. So it produces torque which tends to align $\vec{n}$ with $\vec{B}$.



$$\begin{aligned}\tau &= I a B (b \sin \theta) \\ &= I (a b) B \sin \theta \\ &= I (A) B \sin \theta.\end{aligned}$$

$$F_2 = I b B \sin \theta (90^\circ - \theta)$$

$$\text{same} = I b B \cos \theta = F_1$$

share $\uparrow$ line of action. No torque.

Torque τ tend to rotate the loop.

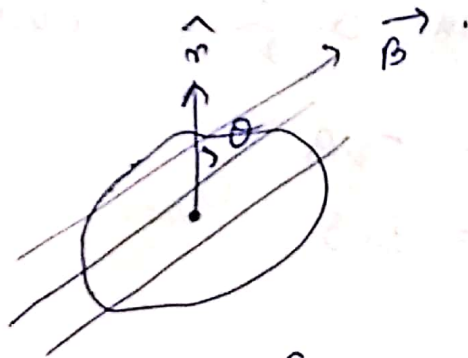
$$\text{If } \theta = 0^\circ$$

$$\text{then } \tau = 0$$

$$\theta = 90^\circ$$

$$\text{then } \tau = I(A)B \cdot (\text{maximum})$$

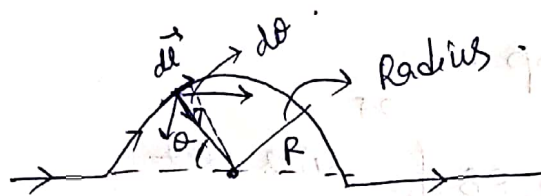
$$A \Rightarrow \text{Area}$$



If there is ~~any~~ ^a loop of any arbitrary shape in a magnetic field. then torque applied on that loop $\tau = \text{current} \times \text{Area of the loop} \times \sin \theta \times \text{magnetic field}$.

We can show it for circular loop also.

4)



half circular loop carrying current I .

Magnetic field outward to the plane of the paper.

We need to calculate the net force on the half circular loop.

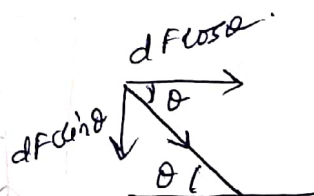
force on the $d\vec{l}$ element.

$$|d\vec{F}| = I dl B \sin 90^\circ$$

$$= I dl B$$

$$= I R d\theta B \quad [dl = R d\theta]$$

$$\begin{aligned} F &= \int I R B d\theta \\ &= I R B \int_0^\pi d\theta \\ &= I R B (\pi) \end{aligned}$$



~~$dF \cos \theta$~~ parallel comp cancelled out
only $\perp$ component remains.

$$\begin{aligned} F &= \int_0^\pi dF \sin \theta \\ &= \int_0^\pi I R d\theta B \sin \theta \\ &= I R B \int_0^\pi \sin \theta d\theta \\ &= I R B [1 - (-1)] \\ &= 2 I R B \end{aligned}$$

For circular loop.

$$F = I R B \int_0^{2\pi} \sin \theta d\theta$$
$$= 0$$

problem

A rectangular loop of wire, supporting mass m , hangs vertically with one end in a uniform magnetic field B , which points into the page in the shaded region. For what current I , in the loop, would the magnetic force upward exactly balance the gravitational force downward.

